LAMC Junior Circle **October 21, 2012**

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Warm-up

Problem 1 *Oleg has locked the handouts for the next Math Circle class in a safe cabinet with a combination lock and forgot the passcode. Each of the five lock dials has ten digits, zero through nine, engraved on it. Oleg remembers that the first digit of his code was not zero and that the last digit was odd. He decides to open the lock by trying all the combinations having the above properties one after another. It takes him 3 seconds to change a combination to the next one. He is willing to turn the dials for 4 hours a day every day. He has a week until the next Math Circle. Would he be able to get the handouts out of the cabinet in time in the worst case of dialling the correct combination the last?*

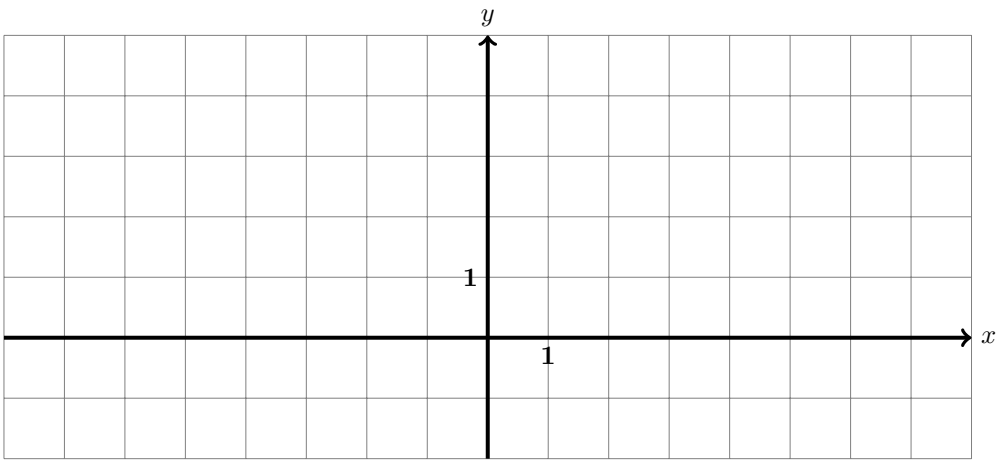
Problem 2 *A prince travelling through a magic land found an enchanted castle guarded by an evil goblin. The goblin told the prince that he had a box with a key to the castle gate, but that it was very dangerous to open the box. The prince immediately accepted the challenge. The goblin presented the young man with three boxes, red, blue, and green. It was written on the red box, "Here is the key." The blue box read, "The green box is empty." The green box had a warning, "There is a poisonous snake in this box." "Ha-ha-ha", laughed the goblin, "it is true that one of these boxes has the key, one is home to a deadly snake and one is empty, but all the labels on the boxes lie. You can only try once. If you open the empty box, you go home empty-handed and if you open the box with the snake, you are dead!" Help the prince to choose wisely.*

Problem 3 *One girl tells another, "There are 25 kids in our Math Circle. Isn't it funny that each kid has 7 friends?" The second girl immediately replies, "This cannot be true." How did she know?*

Functions and graphs, continued

A function is called a *constant function*, if its output is the same for all the inputs. For example, $y = 3$ is a constant function. No matter what x we plug into it, the output is always one and the same, 3.

Problem 4 *Graph the function $y = 3$ on the grid below.*



Recall that we call a function

$$y = ax + b, \quad a \neq 0$$

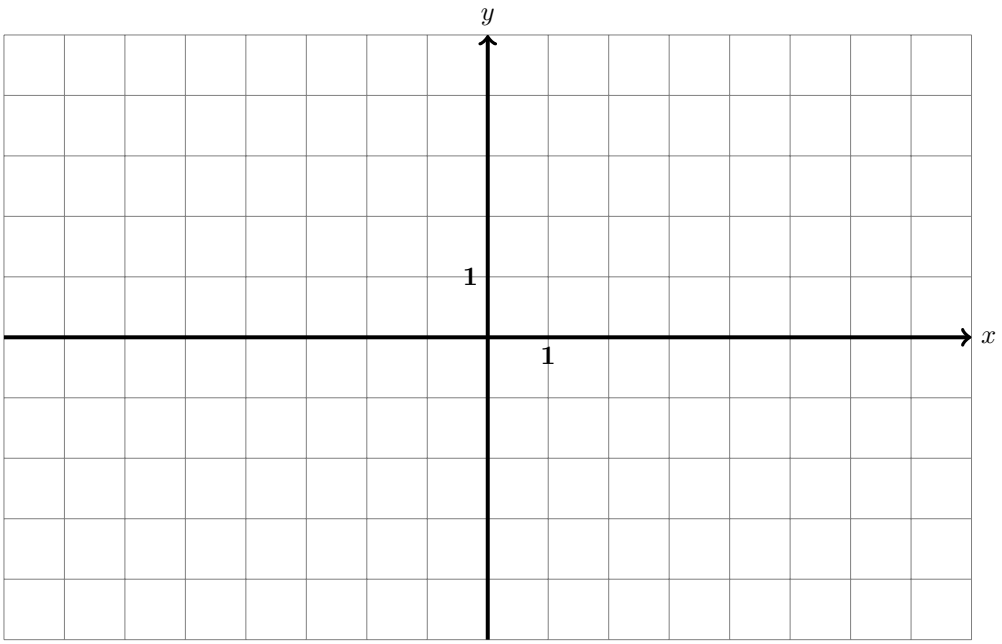
affine if $b \neq 0$ and *linear* if $b = 0$.

Problem 5 *Is the function*

$$y = \frac{x}{2}$$

affine, linear, or neither?

Draw the graph of the function on the grid below.



Recall that the *absolute value* function does not change its non-negative inputs and changes the sign of the negative inputs from $-$ to $+$.

$$|x| = \begin{cases} x & \text{for} \quad x \geq 0 \\ \\ -x & \text{for} \quad x < 0 \end{cases}$$

Problem 6 *Use the graph of the function* $y = x/2$ *from Probelm 5 to draw the graph of the function*

$$y = \left|\frac{x}{2}\right|$$

on the same grid.

Problem 7 *Describe in your own words how one can obtain the graph of the function* $y = |x/2| - 3$ *from the graph of the function* $y = |x/2|$.

Graph the function

$$y = \left|\frac{x}{2}\right| - 3$$

on the grid above.

Problem 8 *Describe in your own words how one can obtain the graph of the function*

$$y = \left| \frac{x-3}{2} \right|$$

from the graph of the function

$$y = \left| \frac{x}{2} \right| .$$

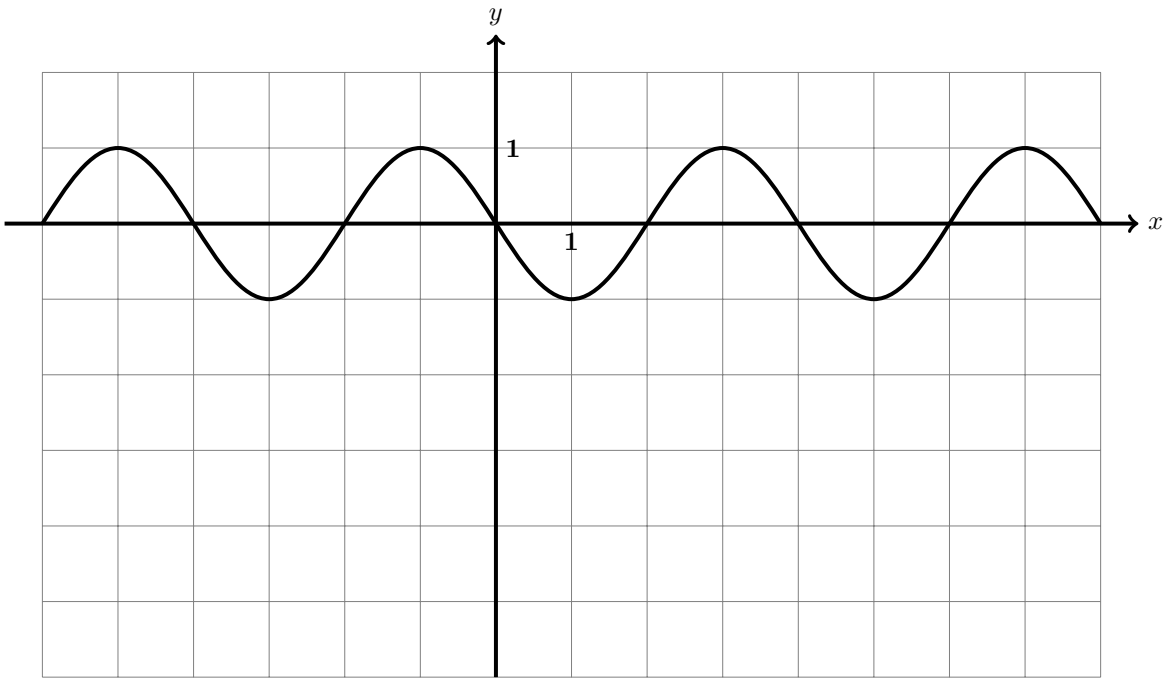
Graph the function

$$y = \left| \frac{x-3}{2} \right|$$

on the grid above.

Note that when you subtract 3 from the function output, y, the graph shifts down by 3. When you subtract 3 from the function input, x, the graph shifts by 3 to the right.

Problem 9 *The following is the graph of the function* $y = f(x)$.



- *Find* $f(5)$.

$$f(5) =$$

- *For what inputs* x *is the output* $f(x) = 1$*? (The waves go to infinity both ways.)*

- *Draw the graph of the function* $y = f(x+1)$ *on the same grid.*

- *Draw the graph of the function* $y = f(x+1) - 4$ *on the same grid.*

Problem 10 *How are the graphs of the functions*

$$y = x^2 - 1 \quad \textit{and} \quad y = (x+2)^2 - 1$$

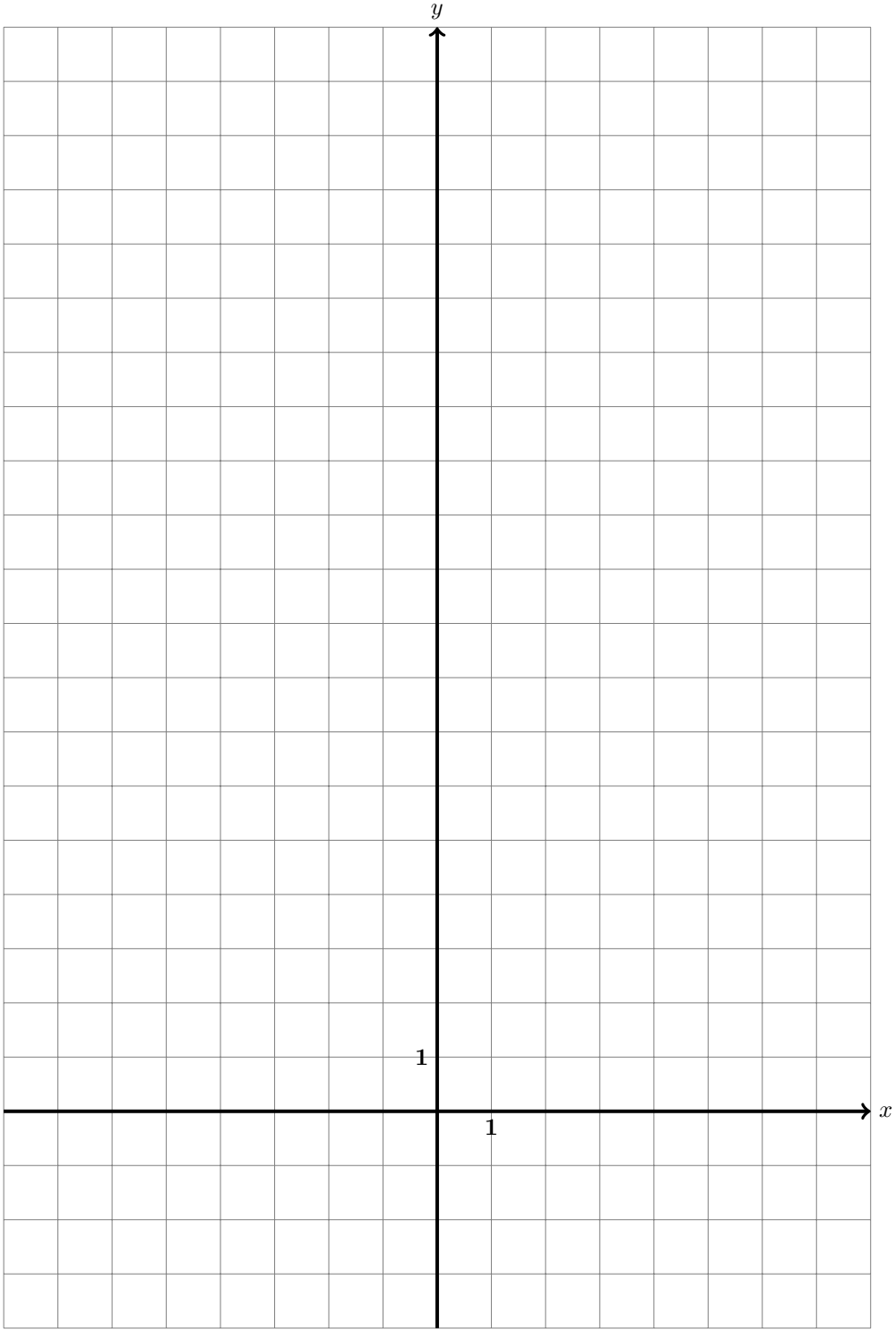
located with respect to each other? Answer the question without graphing either function.

Problem 11 *Fill out the following table and graph the functions*

$$y = x^2 - 1 \quad \textit{and} \quad y = (x+2)^2 - 1$$

on the next page grid.

x	*-3*	*-2*	*-1*	*0*	*1*	*2*	*3*
$x^2 - 1$							
$(x+2)^2 - 1$							


y
x
1
1

Recall that a function of the form

$$y = ax^2 + bx + c, \quad a \neq 0$$

is called a *parabola*.

Problem 12 *Is the function* $y = x^2 - 1$ *a parabola? How about* $y = (x + 2)^2 - 1$*?*

Problem 13 *Use the graphs of the functions* $y = x^2 - 1$ *and* $y = (x + 2)^2 - 1$ *to solve the equation*

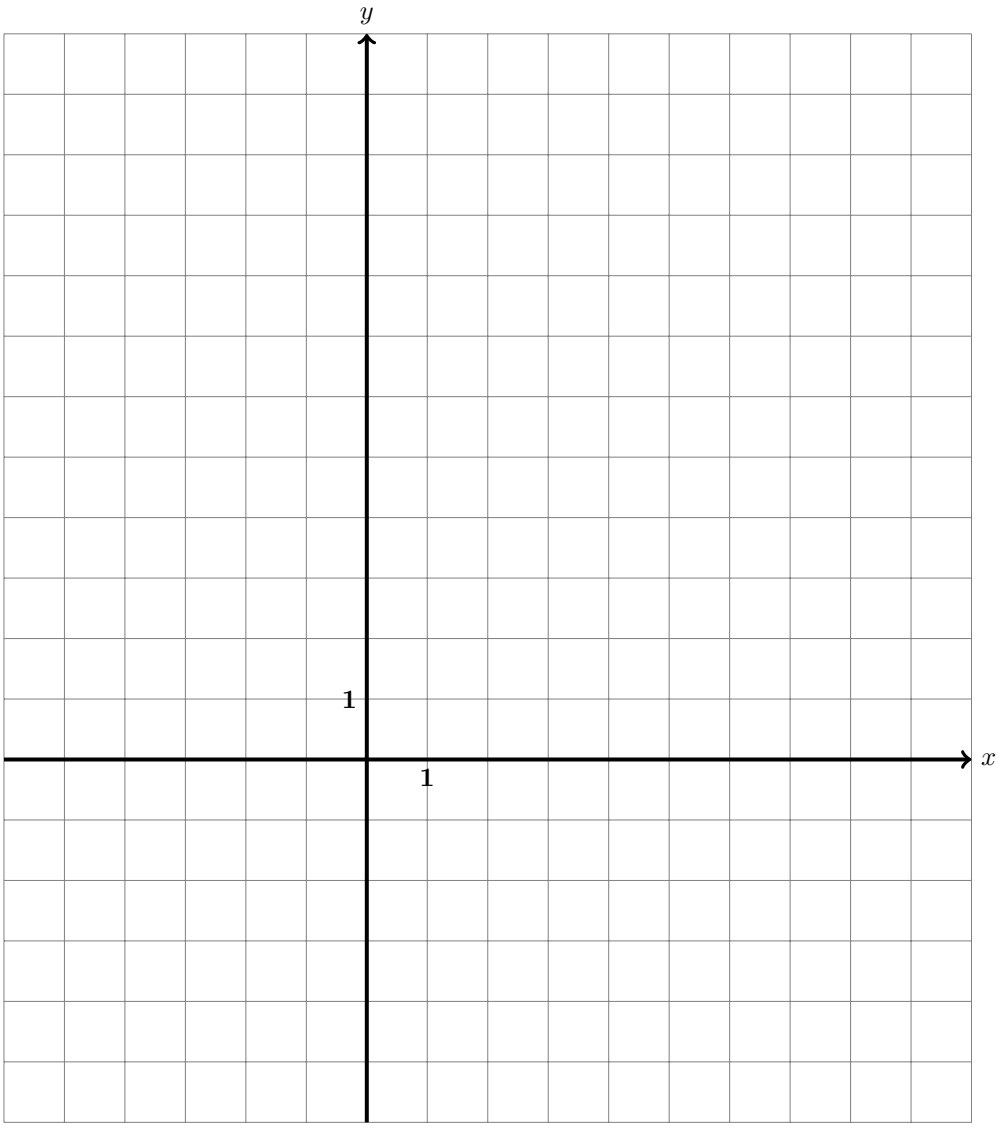
$$x^2 - 1 = (x + 2)^2 - 1.$$

Then solve the equation algebraically to check your answer.

Problem 14 *Solve the following equation.*

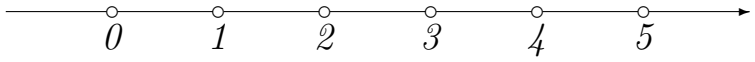
$$x^2 - 8x + 12 = 0$$

Use the below grid to graph the function $y = x^2 - 8x + 12$*, if needed.*



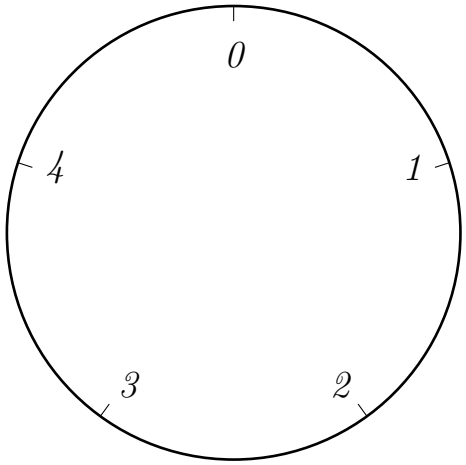
Order on a straight line and a circle

Problem 15 *Looking at the straight number line, put an appropriate sign, $>$, $<$, or $=$, in the boxes between the numbers.*



$0 \square 2 \qquad 0 \square 4 \qquad 3 \square 4$

Now dot the same for the circular mod 5 *number line.*



$0 \square 2 \qquad 0 \square 4 \qquad 3 \square 4$

Can you? Is anything wrong?

If you are finished with all the above and still have time...

Problem 16 *Solve the following cryptarithm.*

$$\begin{array}{cccccc} & C & R & O & S & S \\ + & R & O & A & D & S \\ \hline D & A & N & G & E & R \end{array}$$