Intermediate 1

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What is Probability?

1 What is Probability

A standard 52-card poker deck consists of 4 suits: hearts (\blacklozenge), diamonds (\blacklozenge), spades (\blacklozenge), and clubs (\blacklozenge). Each suit contains number cards from 1 to 10, along with three face cards: Jack (J), Queen (Q), and King (K). We are going to ignore the Joker cards, so a deck contains exactly 52 cards.

Problem 1 *How many hearts* (♥) *are there in a deck of cards?*

Problem 2 What is the proportion of hearts among all cards in a deck? Write this as a fraction, then turn it into percentage notation.

Problem 3 Suppose you draw one card in random from the deck. What is the probability that the card you drawn is a heart?

2 Probability in Math

A probability, also known as a chance, is a number showing how likely some event is to happen. Let us call the event X. Then the probability of X taking place,

 $P(X) = \frac{\text{The number of the outcomes such that } X \text{ happens.}}{\text{The number of all the possible outcomes.}}$

Note that by its very definition, $0 \le P(X) \le 1$.

In the following problems, we will be flipping a coin. Let us denote as H the event of the coin landing heads up and let us denote as T the event of the coin landing tails up.

Problem 5 You flip a coin once. What is the probability of getting the tail?

P(T) =

Problem 6 You flip a coin twice. List all the possible outcomes.

Problem 7 You flip a coin twice. What is the probability of getting the tails both times?

P(TT) =

Problem 8 You flip a coin twice. List all the possible outcomes such that the tails occur at least once.

 $X = \{ \}$

What is the chance that you get the tails at least once?

P(X) =

What is the chance that you never get the tails in two flips?

What number do the last two probabilities add up to? Why?

Problem 9 You roll a single die. List all the possible outcomes.

Problem 10 You roll a single die. What is the chance of rolling an even number?

Problem 11 You roll a single die. What is the chance that the number you get does not exceed two?

Problem 12 You roll a single die. What is the chance that the number you get does not exceed five?

Problem 13 You roll a single die. What is the chance that you roll a six? What number do the probabilities from Problems 12 and 13 add up to? Why?

Problem 14 A wooden cube is painted and cut into 1,000 smaller cubes of equal size.

- How many of the little cubes will have three sides painted?
- How many of the little cubes will have two sides painted?
- How many of the little cubes will have one side painted?

• The little cubes are carefully mixed in a non-transparent bag. Once the mixing is finished, you pull out one of them. What is the chance that the cube you pull out has no paint on it?

Problem 15

• How many two-digit numbers are there? (A two-digit number cannot have zero as the first digit.)

• How many squares of two-digit numbers have one as their last digit?

• What is the probability that a square of a randomly taken two-digit number ends up with one?

Problem 16 Create and solve your own probability problem.

If you are finished doing all the above, but there still remains some time...

Problem 17 (Oldaque P. de Freitas)

Two ladies are sitting in a street café, talking about their children. One lady says that she has three daughters. The product of the girls' ages equals 36 and the sum of their ages is the same as the number of the house across the street. The second lady replies that this information is not enough to figure out the age of each child. The first lady agrees and adds that her oldest daughter has beautiful blue eyes. Then the second lady solves the puzzle. Please do the same.