OLGA RADKO MATH CIRCLE: ADVANCED 3

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Worksheet 1:

Definition Modular Arithmetics. For a, b integers and m positive integers, we say that

$$a \equiv b \pmod{n},$$

(read as "a is congruent to b mod m")if there exists some integer m such that $a - b = mn$.

Problem 1.0: Find the smallest non-negative values of x for the following examples.

- $37 \equiv x \pmod 3$
- $37 \equiv x \pmod 4$
- $37 \equiv x \pmod 5$

Solution 1.0:

Problem 1.1
If we have $a \equiv b \pmod{n}$, $c \equiv d \pmod{n}$ and $a \equiv b \pmod{m}$. Which of the following are always true?

- $a + c \equiv b + d \pmod{n}$
- $ac \equiv bd \pmod{n}$
- $a \equiv b \pmod{m+n}$
- $a \equiv b \pmod{mn}$

Solution 1.1

Problem 1.2:
Find all x, such that there exists an integer y satisfying the following equation:

- $xy \equiv 1 \pmod{2}$
- $xy \equiv 1 \pmod{7}$
- $xy \equiv 1 \pmod{10}$

Solution 1.2:

We say that an integer x has a multiplicative inverse (mod m) if there exists an integer y such that

$$xy \equiv 1 \pmod{m}$$

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Problem 1.3: Prove that if x and m have a common prime factor, then x is has no multiplicative inverse $(mod m)$.
Solution 1.3:

Problem 1.4: Let p be a prime number and x an integer not divisible by p. Show that αx is not divisible by p. Then prove that any integer not divisble by p has a multiplicative inverse (mod p).

Is this true when p is not a prime number.

Solution 1.4:

Problem 1.5: Find all the possible values of y in the following equations

- $x^2 \equiv y \pmod 3$
- $x^2 \equiv y \pmod 4$
- $x^2 \equiv y \pmod 5$
- $x^3 \equiv y \pmod 7$

Solution 1.5:

Problem 1.6: Which of the following equations have solutions?

- $x^3 - y^3 \equiv 4 \pmod 7$
- $x^3 + y^3 \equiv 3 \pmod 7$
- $x^2 + 2x - 1 \equiv 0 \pmod 4$

Solution 1.7:

Problem 1.7: Find all the possible integer solutions to the equation $x^2 + y^2 + z^2 = 411$
Solution 1.7:

Problem 1.8: Graph the solutions of the equation $x^2 + y^2 \equiv 1 \pmod{N}$. For the following values of N:

- $N = 3$
- $N = 10$
- $N = 11$

What happens if you take higher values of N?

Solution 1.8:

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