Nonstandard analysis

Nikita and Stepan

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The conduit is available at https://tinyurl.com/ORMCconduit

Analysis of Infinitesimals

Suppose we have two quantities $x$ and $y$ (say, the radius of a circle and its area), linked by a functional relationship: knowing one, we can calculate the other. Let’s change them slightly $x \rightarrow x + \Delta x$, $y \rightarrow y + \Delta y$, preserving the relationship. The ratio $\frac{\Delta y}{\Delta x}$ (more precisely, its limit when the changes are infinitesimally small) is called the derivative of $y$ with respect to $x$ and is denoted $\frac{dy}{dx}$. If the relationship is given by a function $f$, that is, $y = f(x)$, then we talk about the derivative of the function $f$ at the point $x$ and denote this derivative $f'(x)$:

$$f'(x) = \text{st} \left( \frac{f(x + \Delta x) - f(x)}{\Delta x} \right).$$

**Problem 1.** Using this definition, calculate the derivatives of the following functions:

a) $f(x) = x^2$

b) $f(x) = (x + 3)^2$

c) $f(x) = x^2 + 3$

d) $f(x) = \frac{1}{x}$

e) $f(x) = \sqrt{x}$

**Problem 2.** Fill in the blanks, following the example: The speed of a point on a line is the derivative of its (coordinate) with respect to (time).

a) The acceleration of a point on a line is the derivative of (…) with respect to (…).

b) The heat capacity of a body is the derivative of its internal (…) with respect to (…).

c) The current through a capacitor is the derivative of its (…) with respect to (…).

d) The circumference of a circle is the derivative of (…) of this circle with respect to (…).

e) The area of the surface of a sphere is the derivative of its (…) with respect to (…).

f) The slope of the graph is the derivative of (…) with respect to (…).

**Problem 3.** The width of a rectangle is 2 m and grows at a rate of 1 mm/s. The height of the rectangle is 1 m and grows at a rate of 3 mm/s. What are the rates at which the perimeter and area of this rectangle are growing?
**Problem 4.** A point X moves along a unit circle at a unit speed: at time $t$ its coordinates are $(\cos t, \sin t)$. In what direction is its velocity, and what is its value (i.e., what are its coordinates)?

**Problem 5.** A stick AB is sliding down a wall. What is the speed of the point B?