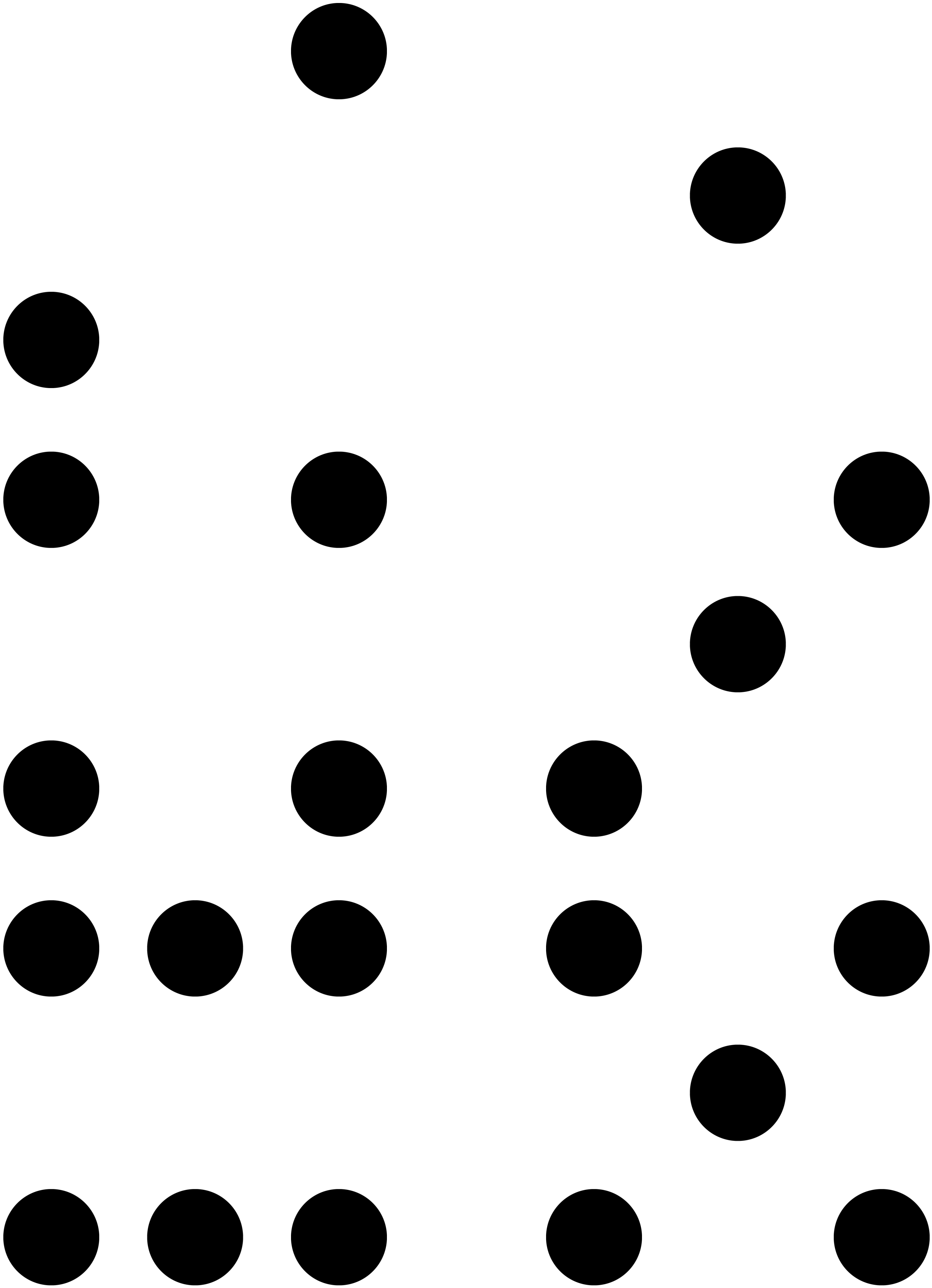


Problem 1 - Algebra - 1 point
AMC 8 2017 Q3

What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$?

Problem 2 - Algebra - 2 points
AMC 8 2018 Q10

The harmonic mean of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4? (Remember that the reciprocal of a number n is $1/n$.)

Problem 3 - Algebra - 3 points
AMC 8 2017 Q14

Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

Problem 4 - Algebra - 4 points
AMC 8 2019 Q16

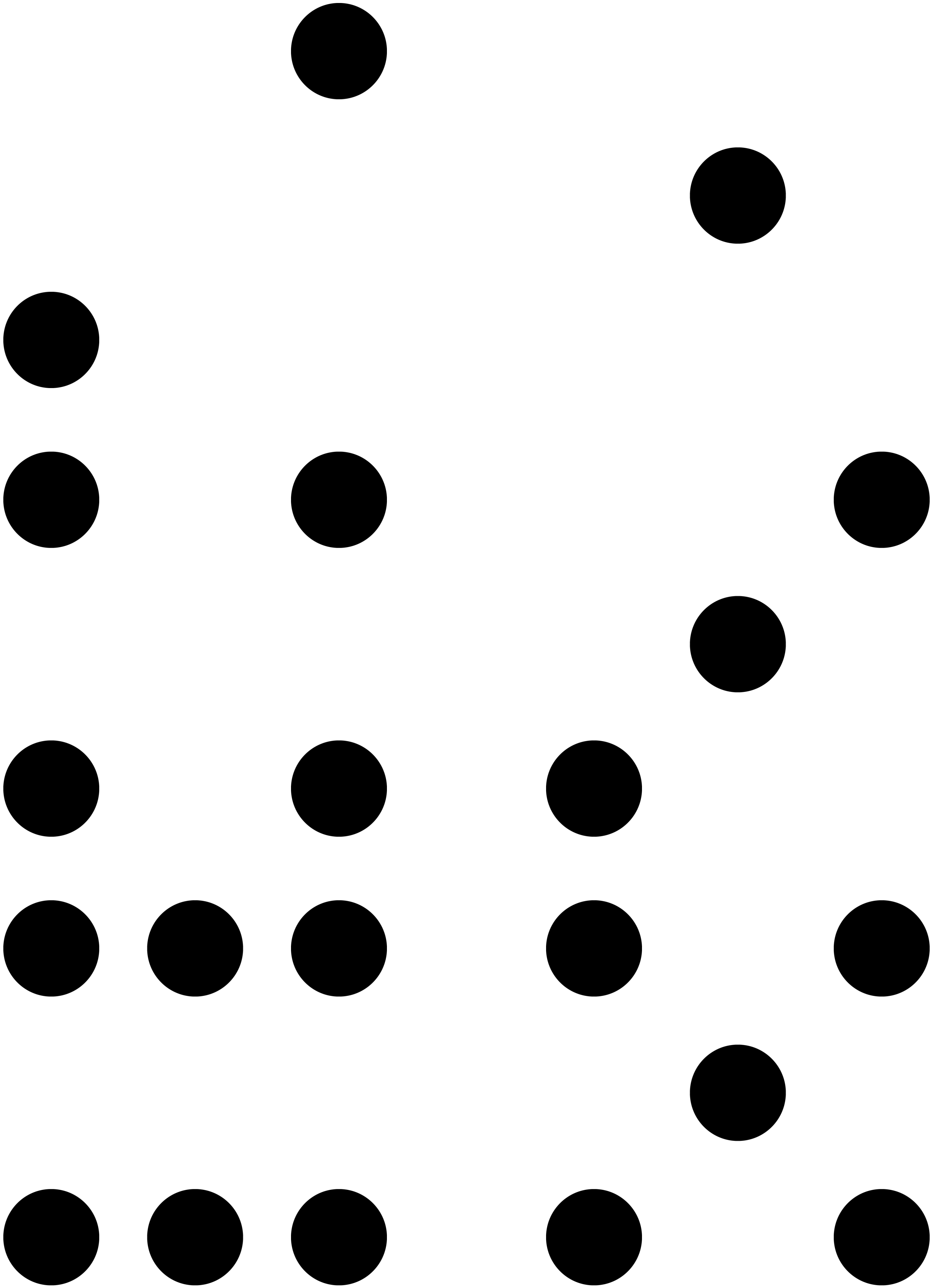
Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip?

Problem 5 - Algebra - 5 points
AMC 8 2017 Q17

Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

Problem 6 - Algebra - 6 points
AMC 8 2005 Q22

A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.



Problem 7 - Arithmetic - 1 point
AMC 8 2004 Q9

The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

Problem 8 - Arithmetic - 2 points
AMC 8 2019 Q20

How many different real numbers x satisfy the equation $(x^2 - 5)^2 = 16$?

Problem 9 - Arithmetic - 3 points
AJHSME 1988 Q21 (Modified)

A number n is added to the set $\{3, 6, 9, 10\}$ to make the mean of the set of five numbers equal to its median. What is the number of possible values of n?

Problem 10 - Arithmetic - 4 points
AMC 8 2007 Q18

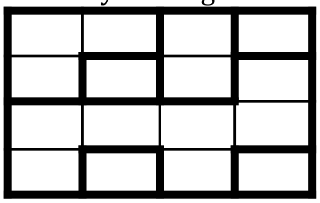
The product of the two 99-digit numbers 303,030,303,...,030,303 and 505,050,505,...,050,505 has thousands digit A and units digit B. What is the sum of A and B? For example, the number 123,456,789 has thousands digit 6 and units digit 9.

Problem 11 - Arithmetic - 5 points
AMC 10A 2005 Q15

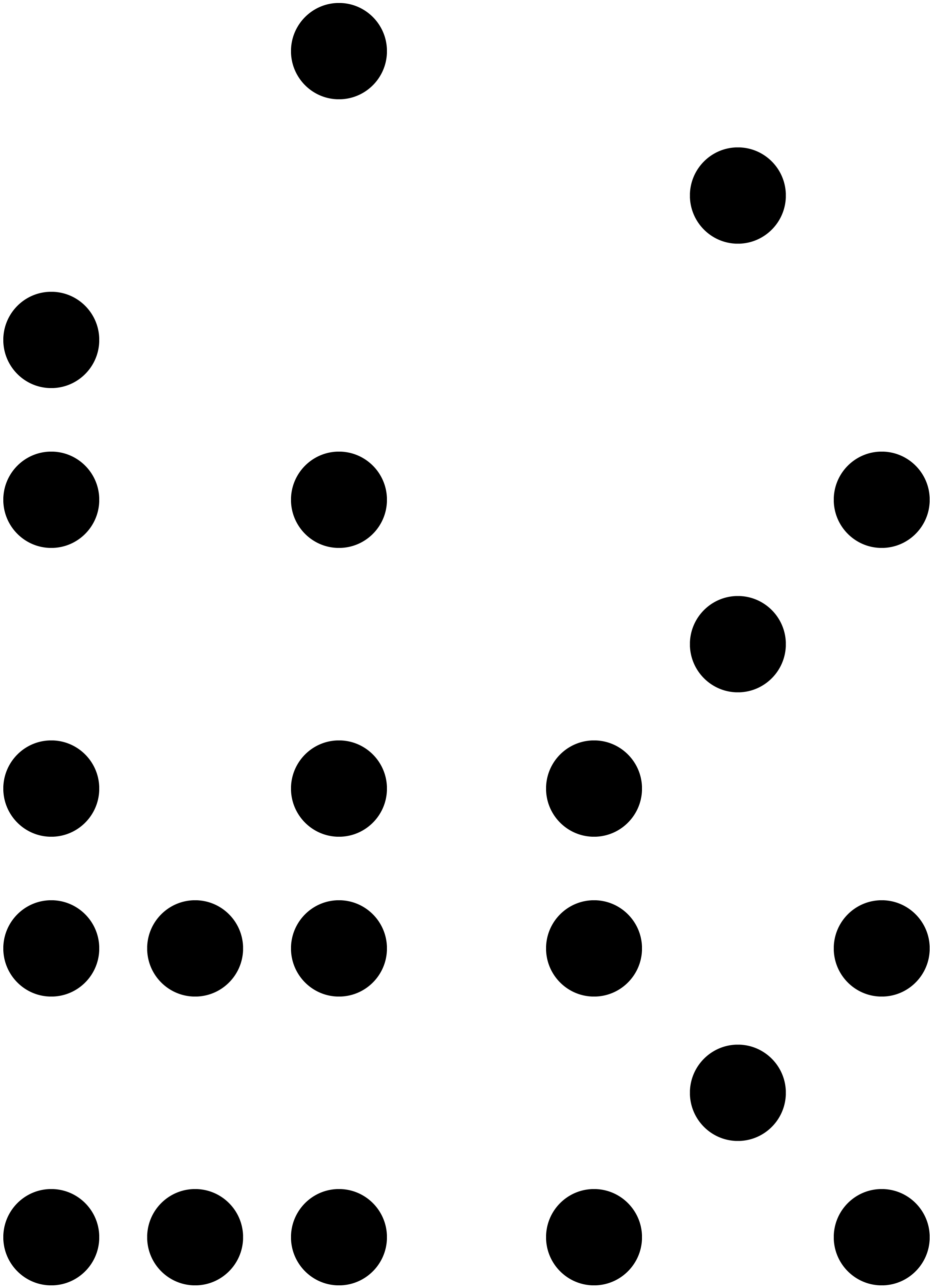
How many positive cubes divide $3! \cdot 5! \cdot 7!$?

Problem 12 - Arithmetic - 6 points
Yan Tao (ytao@math.ucla.edu)

Using the clues, find the 4-digit number that belongs in each row. Cells inside a region (denoted by bolded lines) must all contain the same digit, and each region contains a different digit. (Rows may not begin with 0.)



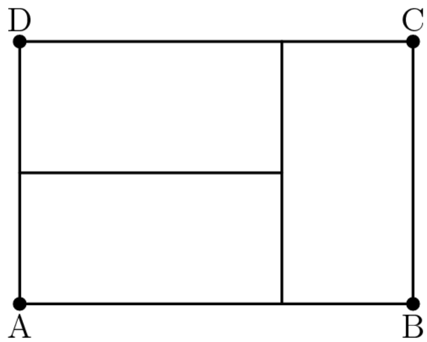
Row 1's digits backwards form a multiple of 64.
Row 2's digits from a decreasing arithmetic progression.
Row 3 is the product of exactly three prime numbers.
Row 4 equals $(n - 2)(n + 2)$ for some integer n.



Problem 13 - Geometry - 1 point

AMC 8 2019 Q2

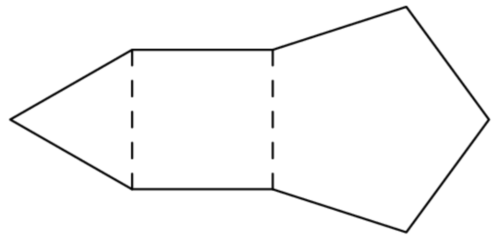
Three identical rectangles are put together to form rectangle $ABCD$. Given that the length of the shorter side of each rectangle is 5 feet, what is the area of rectangle $ABCD$ in square feet?



Problem 14 - Geometry - 2 points

AMC 8 2009 Q9

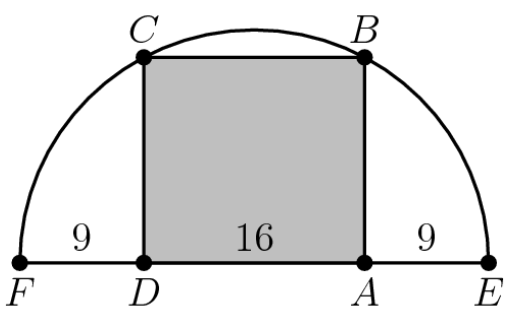
Construct a square on one side of an equilateral triangle. On one non-adjacent side of the square, construct a regular pentagon, as shown. On a non-adjacent side of the pentagon, construct a hexagon. Continue to construct regular polygons in the same way, until you construct an octagon. How many sides does the resulting polygon have?



Problem 15 - Geometry - 3 points

AMC 8 2020 Q18

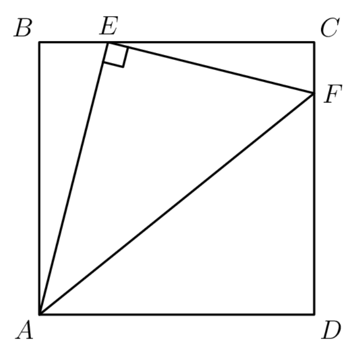
Rectangle $ABCD$ is inscribed in a semicircle with diameter FE as shown in the figure. Suppose $DA = 16$ and $FD = AE = 9$. What is the area of $ABCD$?



Problem 16 - Geometry - 4 points

David Altizio (altizio2@illinois.edu)

Triangle AEF is a right triangle with $AE = 4$ and $EF = 3$. The triangle is inscribed inside square $ABCD$ as shown. What is the area of the square?



Problem 17 - Geometry - 5 points

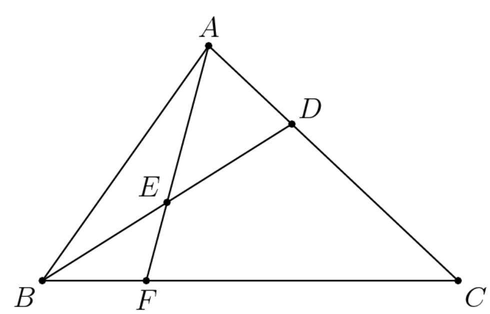
AMC 10B 2012 Q21

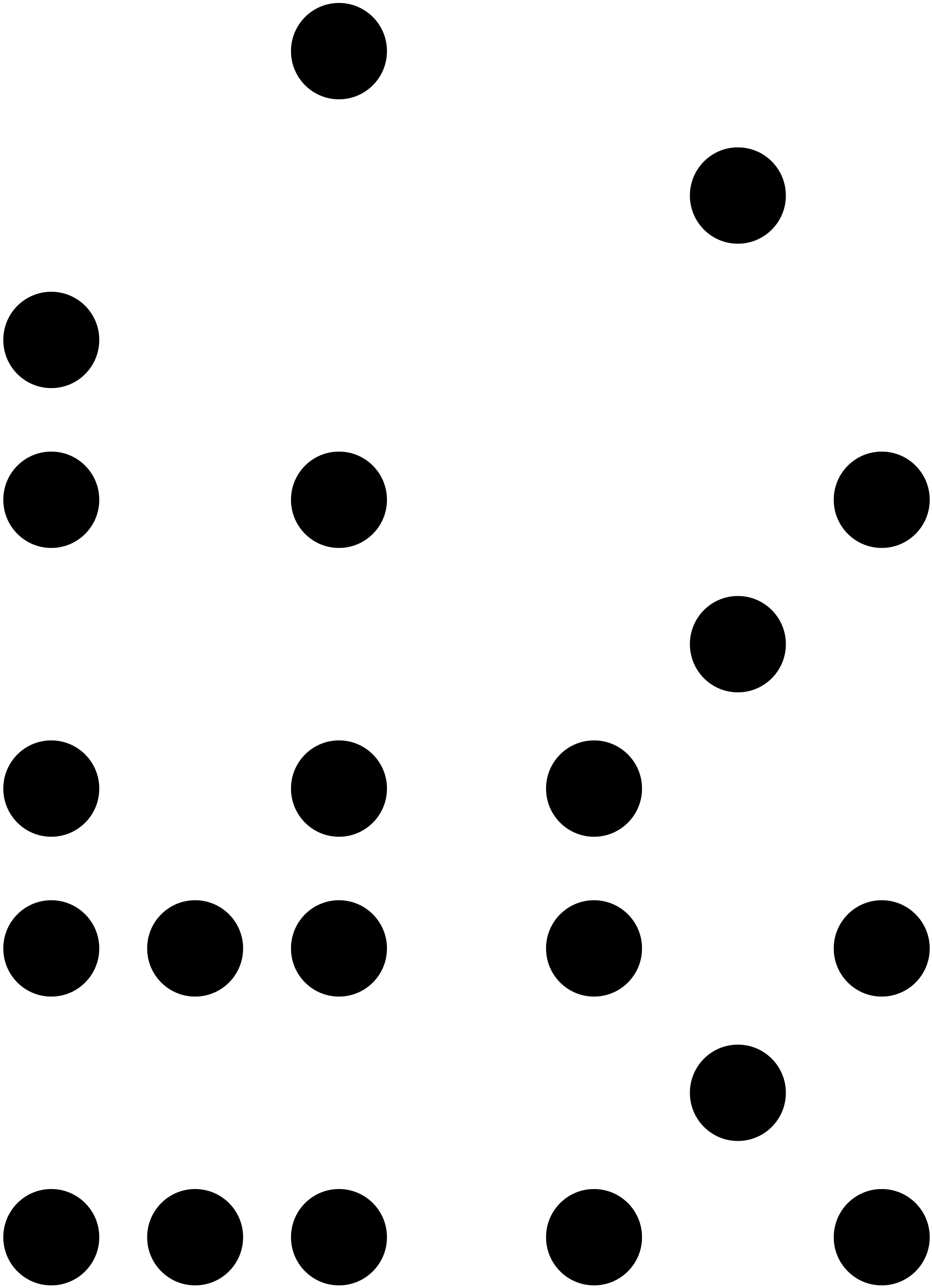
Four distinct points are arranged in the plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

Problem 18 - Geometry - 6 points

AMC 8 2019 Q24

In triangle ABC, point D divides side AC so that $AD: DC = 1: 2$. Let E be the midpoint of BD and let F be the point of intersection of line BC and line AE. Given that the area of triangle ABC is 360, what is the area of triangle EBF?





Problem 19 - Combinatorics and Probability - 1 point
AMC 8 2004 Q2

How many different four-digit numbers can be formed by rearranging the four digits in 2004?

Problem 20 - Combinatorics and Probability - 2 points
AJHSME 1987 Q25

Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. What is the probability that the sum of the two numbers on the balls removed is even?

Problem 21 - Combinatorics and Probability - 3 points
AMC 8 2019 Q18

The faces of each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?

Problem 22 - Combinatorics and Probability - 4 points
AMC 10B 2009 Q11 (Modified)

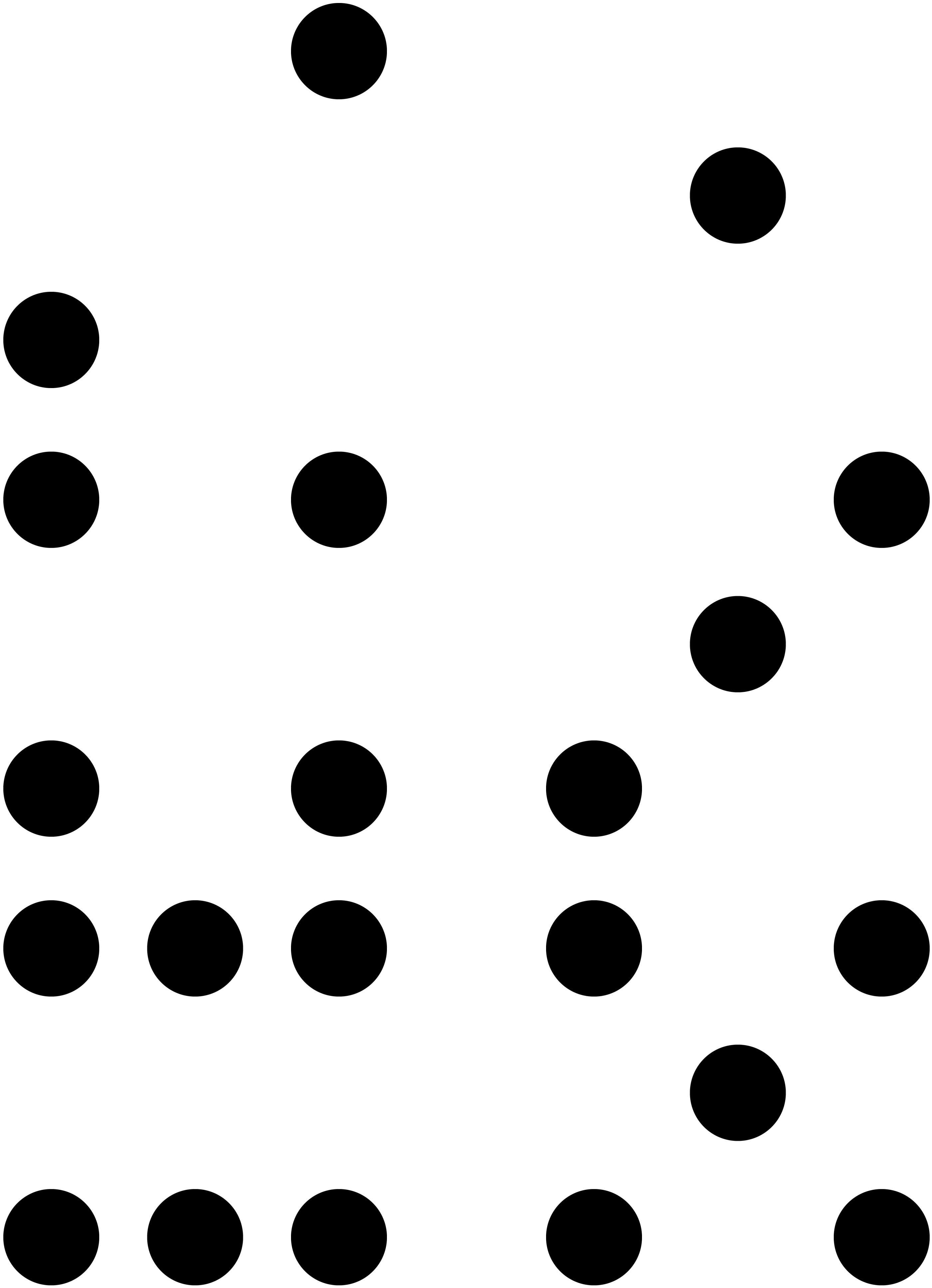
How many 7-digit palindromes (numbers that read the same forwards and backwards) that do not have more than 2 occurrences of any digit are there?

Problem 23 - Combinatorics and Probability - 5 points
AMC 8 2019 Q25

Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the people has at least 2 apples?

Problem 24 - Combinatorics and Probability - 6 points
AMC 8 2018 Q23

From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



Problem 25 - Boolean Algebra - 1 point

Create a truth table for the following Boolean algebra expression.

$R\neg S + RS + \neg RS$

Problem 26 - Boolean Algebra - 2 points

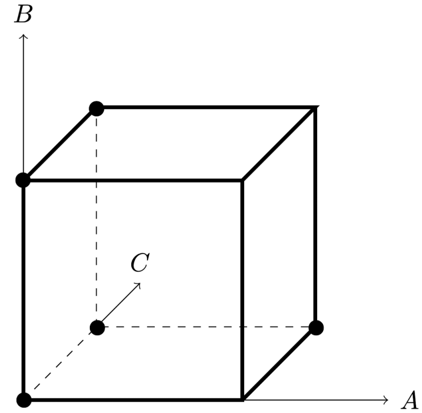
Fully simplify the following Boolean algebra expression.

$\neg X + X(X + \neg Y)(Y + \neg Z)$

Then, write out the truth table for the simplified expression.

Problem 27 - Boolean Algebra - 3 points

Write down the expression in full disjunctive normal form that corresponds to the marked vertices of the cube, and use the cube to simplify.



Problem 28 - Boolean Algebra - 4 points

By means of algebraic manipulation, prove the consensus theorem. That is, show:

$TU + \neg TV + UV = TU + \neg TV$

Problem 29 - Boolean Algebra - 5 points

In addition to the operators AND, OR, and NOT, there is an operator NAND that is represented by the symbol ↑. The expression $A \uparrow B$ means $\neg(AB)$.

Rewrite the following expression using only ↑ operators.

$A\neg B + CD$

Problem 30 - Boolean Algebra - 6 points

Geometrically represent a 7-variable Boolean expression in full disjunctive normal form. Answer can be verbally explained to the instructors.