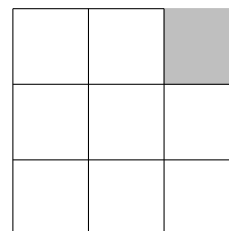
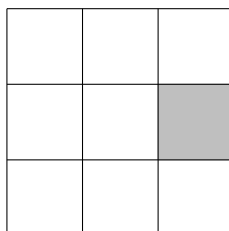
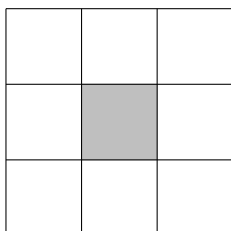


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Warm-up

Problem 1 *Two pieces were used to make each of the three white shapes below. (The gray square is missing.) Draw the pieces.*



Problem 2 *Write three numbers in a row so that the sum of any two consecutive numbers is even, but the sum of all the three of them is odd.*

Problem 3 *The mother has bought a box of sugar in cubes. First, the children have eaten the top layer of 35 cubes, then the right hand side layer of 21 cubes, and finally they ate the front layer. How many sugar cubes were left in the box?*

Functions and graphs

For a given formula $f(x)$ that assigns some value $y = f(x)$ to all the input values of x , the *graph* of the function $y = f(x)$ is the set of all the points $(x, y(x))$ in the plane.

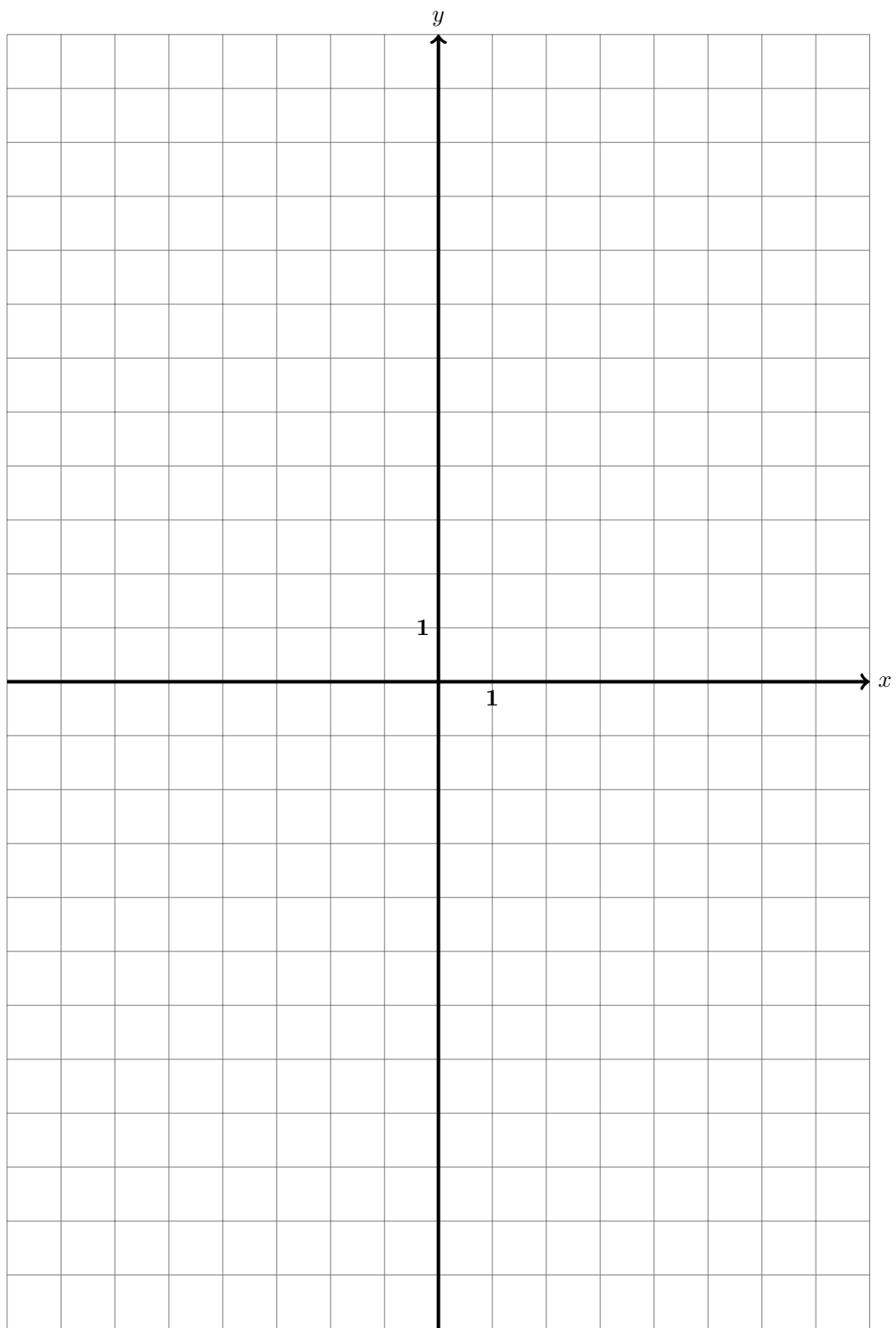
Problem 4 *A function is given by the following formula.*

$$y = 3x - 1 \tag{1}$$

Fill out the below table.

x	-3	-2	-1	0	1	2	3
y							

- *Mark the points (x, y) from the table on the next page grid.*
- *What shape does the graph of the function $y = 3x - 1$ form?*
- *Find a point on the shape not listed in the table above. Write down the coordinates of the point in the space below. Check if the relation 1 is satisfied.*



Problem 5 Draw the graph of the function

$$y = 3x + 1$$

on the grid above. Construct a table of values, if needed.

x							
y							

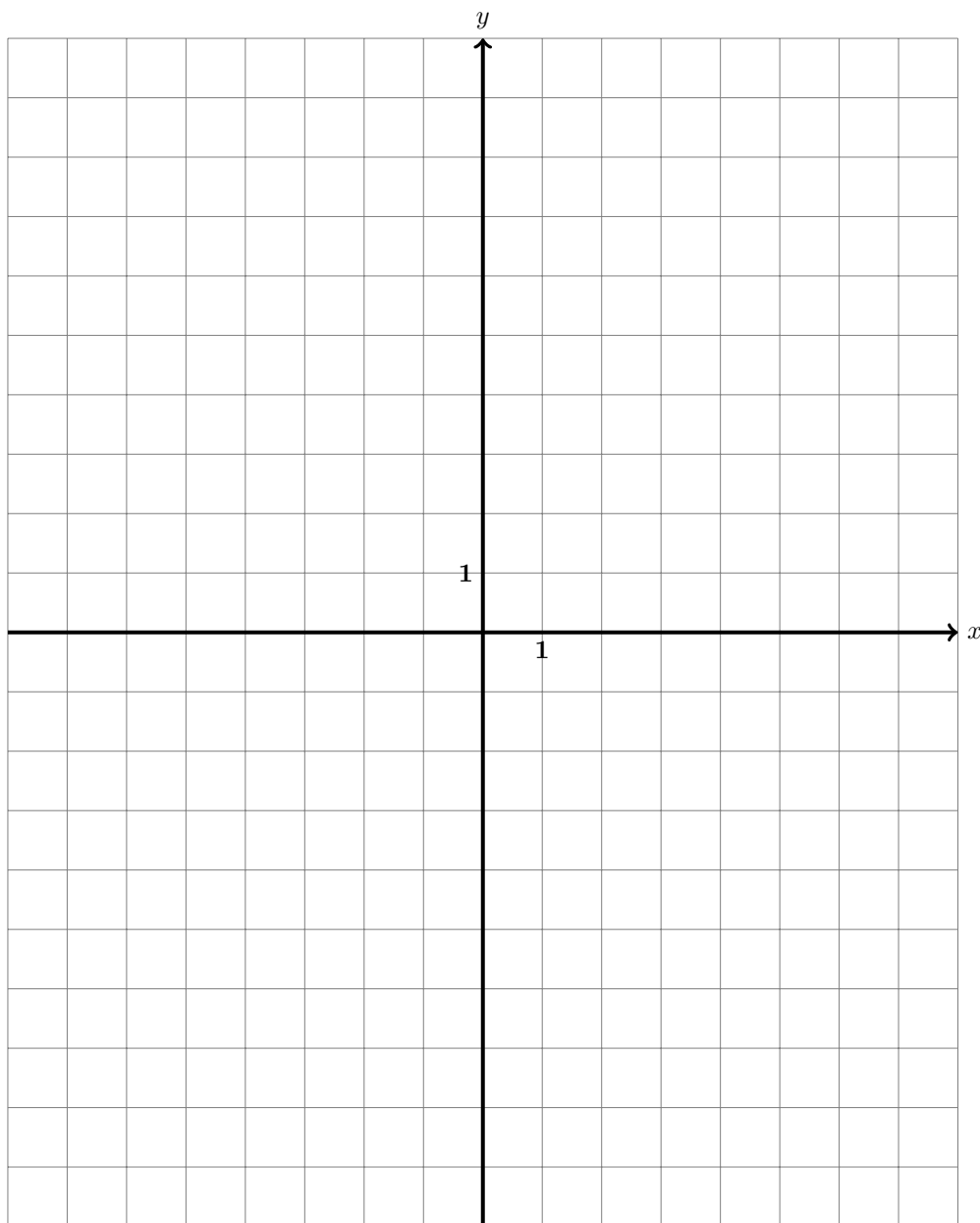
Take a good look at the two graphs, then answer the following question. How can you obtain the graph of the function $y = 3x + 1$ from the graph of the function $y = 3x - 1$?

Problem 6 Guess the formula relating x and y in the following table.

x	-3	-2	-1	0	1	2	3
y	6	4	2	0	-2	-4	-6

$$y =$$

Draw the graph of the function on the grid below.



What is $y(23)$?

$$y(23) =$$

A function of the form

$$y = ax + b, \quad a \neq 0 \tag{2}$$

is called *affine* for $b \neq 0$ and *linear* for $b = 0$.

Warning: There exists a terminology discrepancy between the school teachers and working mathematicians. The former call “linear” any function of the form 2.

The following function

$$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

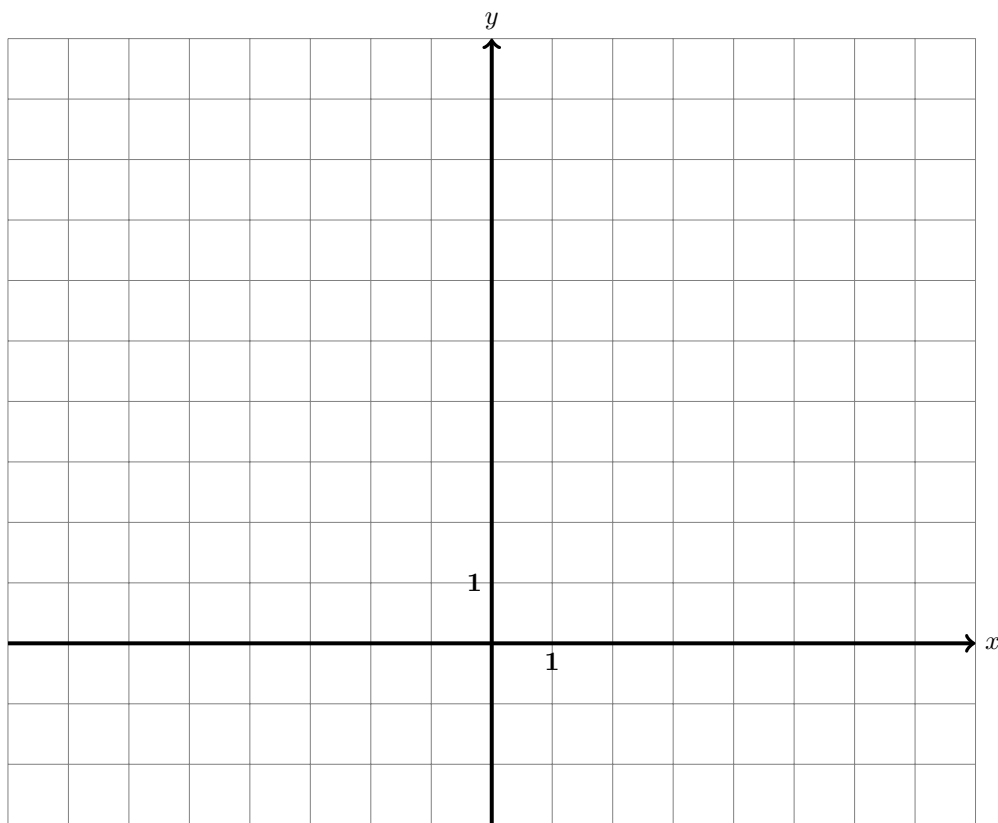
is called the *absolute value of x* .

Problem 7 Make a table of values for the function $y = |x|$.

x	-3	-2	-1	0	1	2	3
y							

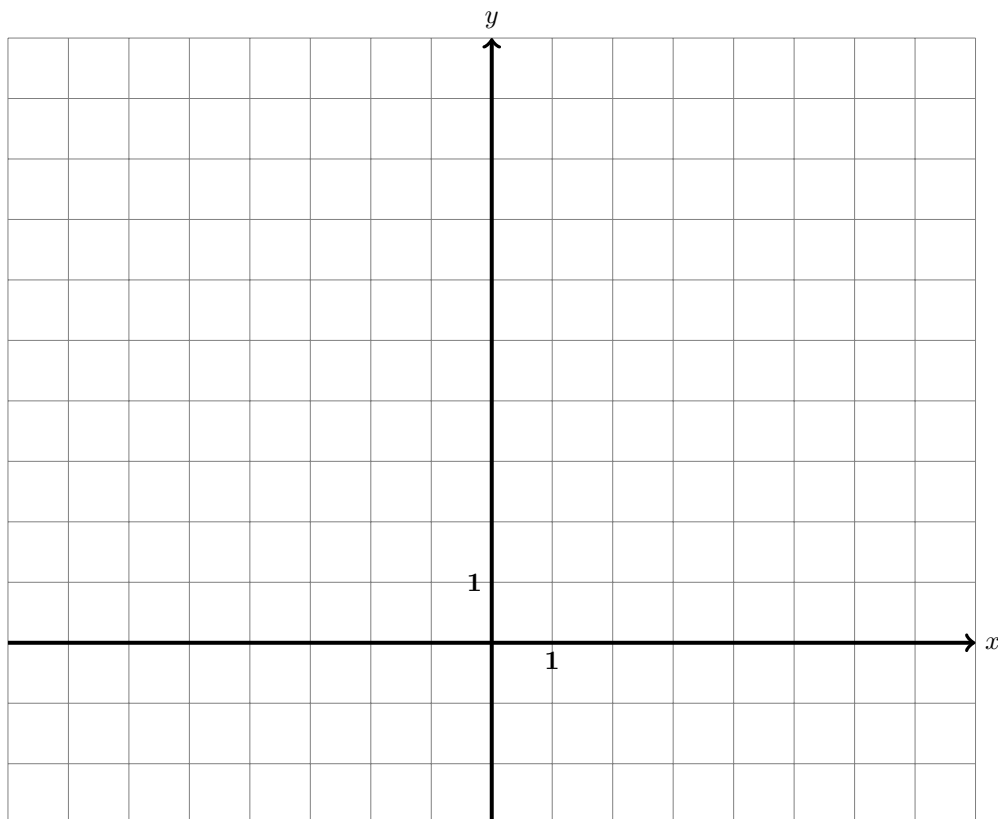
Can a value of the function $y = |x|$ be negative? Why or why not?

Problem 8 Graph the function $y = |x|$ on the grid below.



Problem 9 Graph the functions $y = |x - 2|$ and $y = |x| - 2$ on the above grid. How can they be obtained from the function $y = |x|$?

Problem 10 Graph the function $y = ||x - 2| - 2|$ on the grid below. Hint: split the problem into steps.

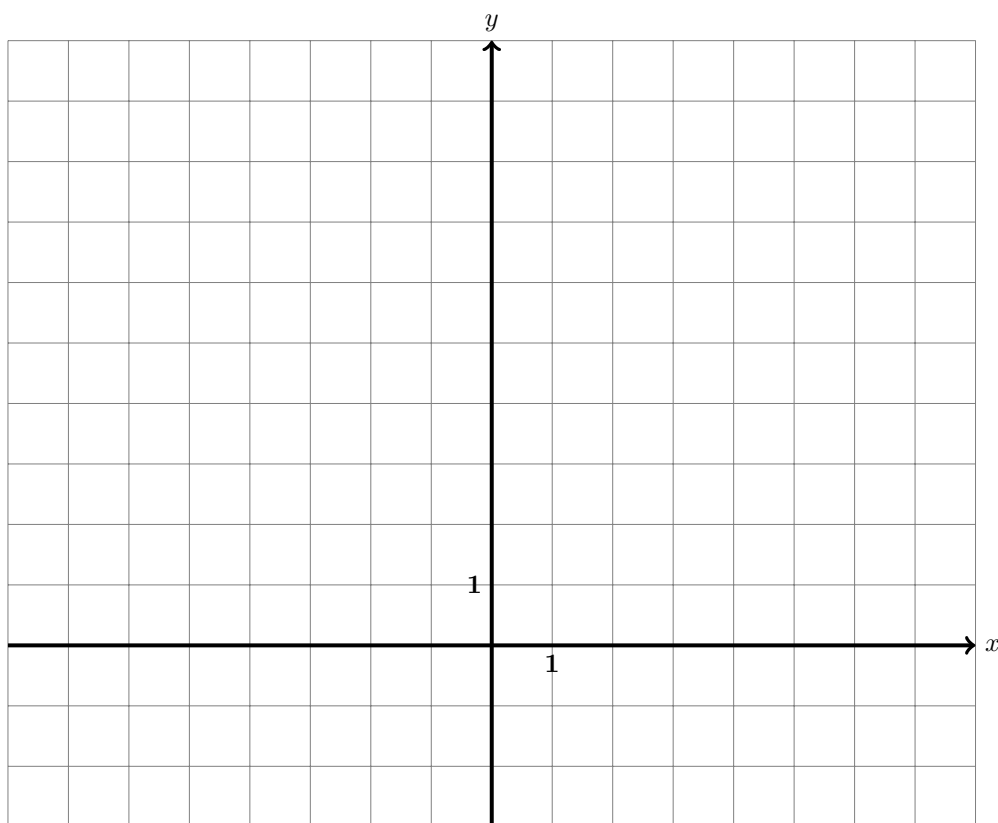


Problem 11 For the function $y = \underbrace{|\dots ||x - 2| - 2| \dots - 2|}_{100 \text{ times}}$, find the value $f(200)$.

$$f(200) =$$

Problem 12 *What is the output of the function $y = 5$ for any input value of x ?*

Draw the graph of the function $y = 5$ on the grid below.

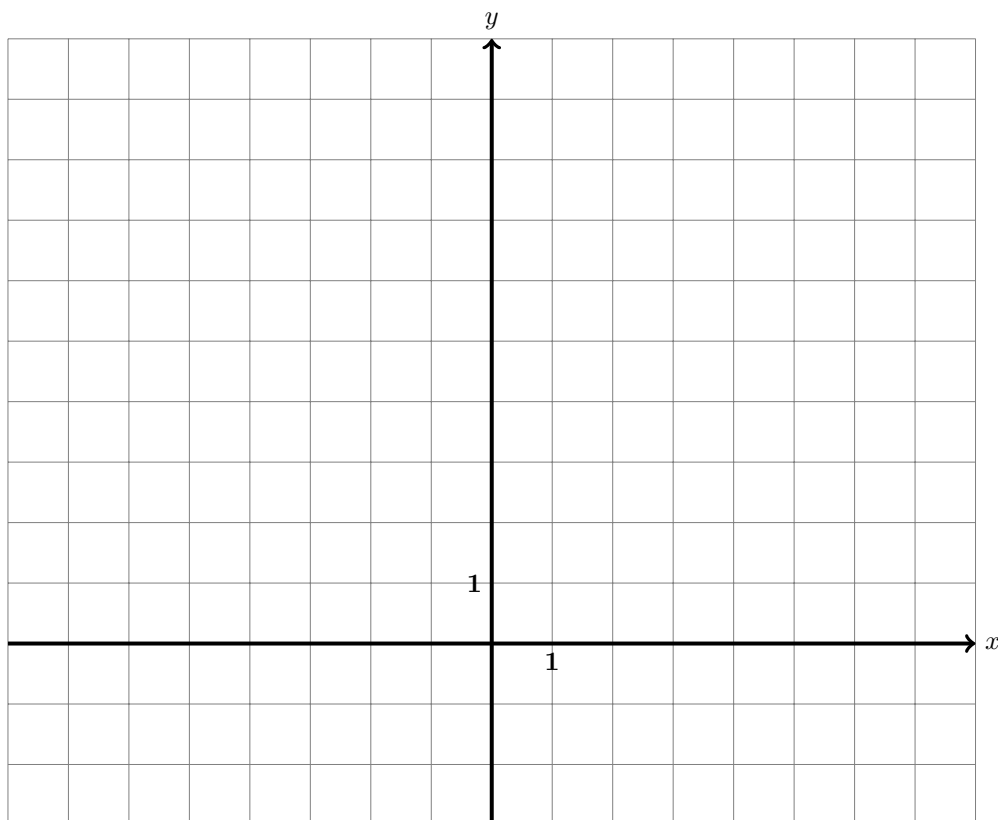


Graph the equation $x = 3$ on the grid above. Is the graph a graph of a function of x ?

Problem 13 *Make a table of values for the function $y = x^2$.*

x	-3	-2	-1	0	1	2	3
y							

Draw the graph of the function $y = x^2$ on the grid below.

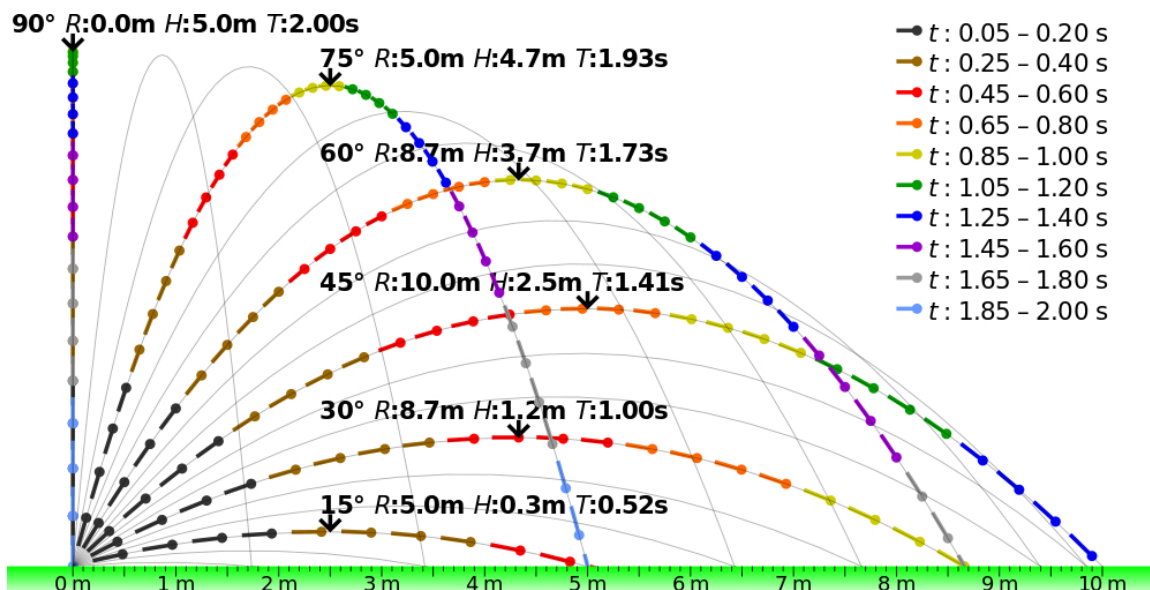


Problem 14 Draw the graphs of the functions $y = x^2 - 1$ and $y = (x - 1)^2$ on the grid above. How can the graphs be obtained from the graph of the function $y = x^2$?

A function of the form

$$y = ax^2 + bx + c, \quad a \neq 0 \quad (3)$$

is called a *parabola*. This function is very important for physics. In the absence of an atmosphere, an object thrown away from the surface of a planet at an angle different from the right angle (and slow enough) moves along a parabola until it hits the ground.



Downloaded from <http://en.wikipedia.org/wiki/Trajectory>

Let us call x the horizontal distance of the thrown object from the initial point and let us call y its height above the ground.

Problem 15 *A thrown rock moves according to the following equation.*

$$y = x - \frac{x^2}{10} \tag{4}$$

- *Is this an equation of a parabola? Why or why not?*

- *How far from the starting point will the rock hit the ground?*

- *Find the graph of the function (4) on the picture above. Use the graph to determine the maximal height of the rock during the flight.*

$$y_{max} =$$

- *At what angle was the rock thrown?*

A bit more of clock arithmetic

Problem 16

$$\frac{1}{2^{34}} \equiv \quad \text{mod } 5$$

$$2^{-34} \equiv \quad \text{mod } 5$$