

GEOMETRY

MATH CIRCLE (ADVANCED) 10/14/2012

1) Prove that the length of the median AM in triangle ABC is greater than $\frac{AB+AC-BC}{2}$.

2) Prove that you can form a triangle from segments with length a, b, c if and only if there are positive numbers x, y, z such that $a = x + y, b = y + z, c = x + z$.

3) The centers of three non-intersecting circles lie on the same straight line. Prove that if a fourth circle touches all three given circles, then its radius is greater than at least one of the given three.

4) Prove that if you can form a triangle from segments of length a, b, c , then you can form a triangle from segments of length $\sqrt{a}, \sqrt{b}, \sqrt{c}$.

5) The lengths of the sides of a convex quadrilateral are a, b, c, d (listed clockwise). Prove that the area of the quadrilateral does not exceed:

a) $\frac{ab+cd}{2}$.

b) $\frac{(a+b)(c+d)}{4}$.

6) Points K, L, M, N are the midpoints of the sides of the convex quadrilateral $ABCD$. Prove that $2 \times \text{Area}(KLMN) = \text{Area}(ABCD)$.

7) If all sides of a triangle are longer than 1000 inches, can its area be less than one square inch?

8) Triangle ABC is given. Point A_1 lies on segment BC extended beyond point C , and $BC = CA_1$. Points B_1, C_1 are constructed in the same way (extending AC past A and AB past B). If $\text{Area}(ABC) = 1$, what is $\text{Area}(A_1B_1C_1)$?

9) Prove that if two convex quadrilaterals have the same midpoints for all their sides, then their areas are equal.

10)* Find the midpoint of a segment

a) using a compass only.

b) using only a two-sided ruler (with two parallel sides), whose width is less than the length of the segment.

Some problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”