ORMC AMC 10-12A final contest

June 2023

1 Individual Problems

Instructor note: **Problems given 2 at a time, 10 minutes per each 2 problems**

1. Doug and Dave shared a pizza with 8 equally-sized slices. Doug wanted a plain pizza, but Dave wanted anchovies on half the pizza. The cost of a plain pizza was 8 dollars, and there was an additional cost of 2 dollars for putting anchovies on one half. Dave ate all the slices of anchovy pizza and one plain slice. Doug ate the remainder. Each paid for what he had eaten. How many more dollars did Dave pay than Doug?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

   (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

3. An equilateral triangle of side length 10 is completely filled in by non-overlapping equilateral triangles of side length 1. How many small triangles are required?

   (A) 10 (B) 25 (C) 100 (D) 250 (E) 1000

4. A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

5. Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?

(A) $\frac{12+3\sqrt{3}}{4}$ **(B)** $\frac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\frac{6+3\sqrt{3}}{2}$ **(E)** 6

6. All the numbers $1,2,3,4,5,6,7,8,9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

7. Let S be a set of 6 integers taken from $\{1,2,\ldots,12\}$ with the property that if a and b are elements of S with $a<b$, then b is not a multiple of a. What is the least possible value of an element in S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

8. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

9. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

(A) 6 **(B)** 9 **(C)** 13 **(D)** 16 **(E)** 18

10. Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of walls of sizes 4 and 2 can be changed into any of the following by one move: $(3,2),(2,1,2),(4),(4,1),(2,2)$, or $(1,1,2)$.

    Arjun plays first, and the player who removes the last brick wins. For which starting configuration is there a strategy that guarantees a win for Beth?

(A) $(6,1,1)$ **(B)** $(6,2,1)$ **(C)** $(6,2,2)$ **(D)** $(6,3,1)$ **(E)** $(6,3,2)$

2 Relay Problems

Instructor note: **Using Standard ARML Relay rules, but with time doubled– 6 minutes for first submission, 12 for second submission**

2.1 Relay 1

1. Compute the number of ordered pairs (x, y) of positive integers satisfying $x^2 - 8x + y^2 + 4y = 5$
2. Let $T = TNYWR$. Let $k = 21 + 2T$. Compute the largest integer n such that $2n^2 - kn + 77$ is a positive prime number.
3. Let $T = TNYWR$. In triangle ABC, $BC = T$ and $m\angle B = 30°$. Compute the number of integer values of AC for which there are two possible values for side length AB.

2.2 Relay 2

1. If A, R, M, L are positive integers such that $A^2 + R^2 = 20$ and $M^2 + L^2 = 10$, compute the product $A \cdot R \cdot M \cdot L$.
2. Let $T = TNYWR$. A regular n-gon is inscribed in a circle. P and Q are consecutive vertices of the polygon, and A is another vertex of the polygon. If $m\angle AQP = m\angle APQ = T \cdot m\angle QAP$, compute the value of n
3. Let $T = TNYWR$. Compute the last digit, in base 10, of the sum
$$T^2 + (2T)^2 + (3T)^2 + \cdots + (T^2)^2.$$