LAMC Junior Circle **October 7, 2012**

Oleg Gleizer
oleg1140@gmail.com

Warm-up

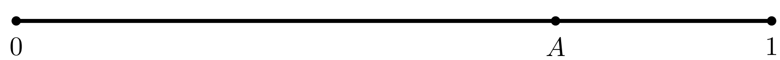
Problem 1 *(Oldaque P. de Freitas Puzzle)*

Two ladies are sitting in a street café, talking about their children. One lady says that she has three daughters. The product of the girls' ages equals 36 and the sum of their ages is the same as the number of the house across the street. The second lady replies that this information is not enough to figure out the age of each child. The first lady agrees and adds that her oldest daughter has beautiful blue eyes. Then the second lady solves the puzzle. Please do the same.

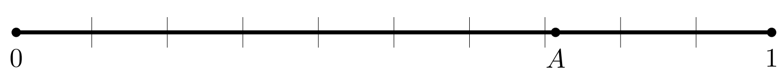
Coordinates

Coordinates are ordered lists of numbers marking points in space, on a surface, or curve. In the latter case, there is only one coordinate, which can be considered as a list with one element.

Let us first introduce coordinates on a straight line segment. Let us mark the left end of the segment with the number 0 and the right end with the number 1. Let us pick a point on the segment and find out what number, or coordinate, corresponds to the point.



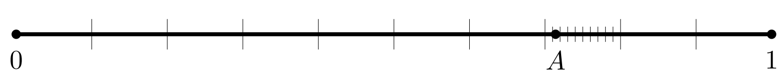
In order to do that, let us split the segment into ten equal parts.



If one of the marks hits the point, then we know its coordinate. In our case, this doesn't happen. We don't get the coordinate right away. We get an estimate instead.

$$0.7 < A < 0.8$$

Let us further split the segment between the points 0.7 and 0.8 into ten equal parts, each being 100 times shorter than the original segment.



This gives a better estimate for our point.

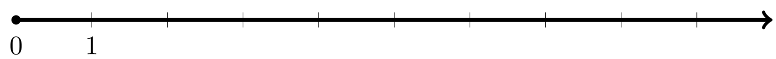
$$0.71 < A < 0.72$$

Since A doesn't hit the mark again, we split the segment between the points 0.71 and 0.72 into ten equal parts one more time. Now, each of the parts is 1,000 times shorter than the original segment. In the end, either A hits the mark and we get a final decimal representation for it, or A never hits the mark. In the latter case, A gets represented by an infinite decimal number.

In our example, $A = 5/7$.

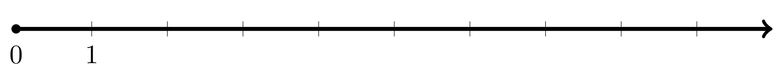
Problem 2 *Find the decimal representation of* A*. Is it finite or infinite?*

Problem 3 *Point B has the coordinate* 4.5. *Mark it on the straight line below.*



Problem 4 *Point C has the coordinate* 8.333..., *the threes going all the way to infinity. Can you represent the number in the form $8\frac{p}{q}$ where p and q are integers such that $1 \leq p < q$?*

Mark point C on the straight line below.



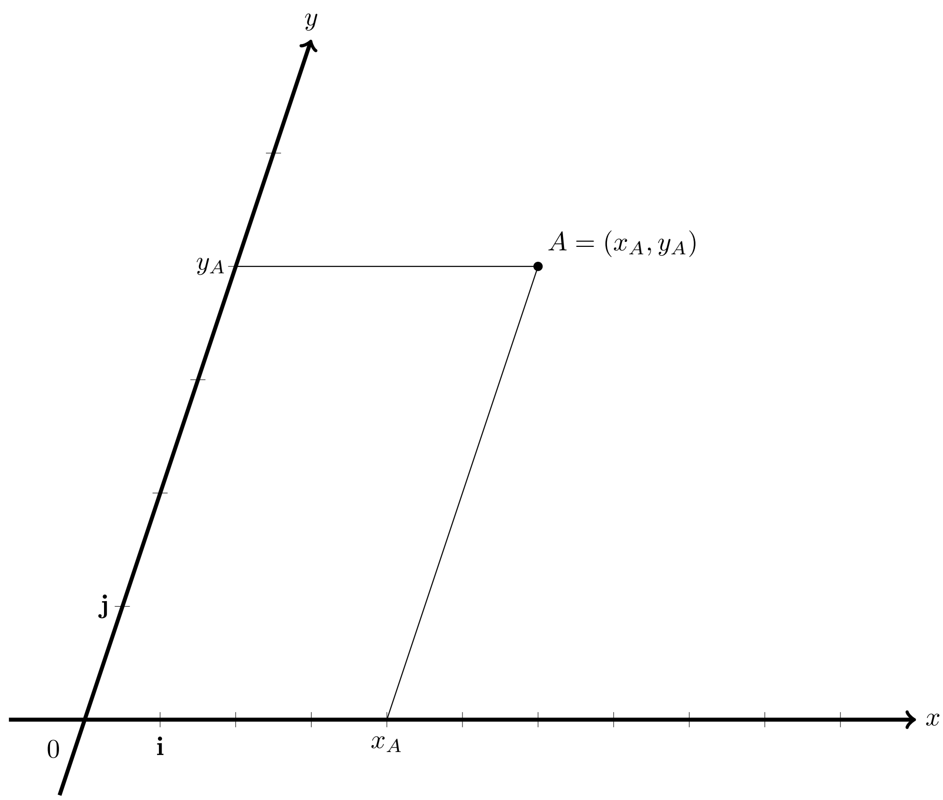
As you can see, introducing coordinates on a straight line is the same thing as turning it into the number line. Each point gets uniquely represented by a (real) number and, the other way around, each (real) number gets uniquely represented by a point on the line. We need two numbers for a 2D surface. Let us take a look at the simplest one, the Euclidean plane.

To mark every point in the plane with a pair of numbers, let us choose the origin, point O, and two different straight lines passing through it, Ox and Oy. Let us choose the unit steps, $\mathbf{i}$ and $\mathbf{j}$ along Ox and Oy respectively (see the picture below).

Recall that a *postulate*, or *axiom*, is a mathematical statement accepted as self-evident and not requiring a proof. We will need the following as a tool.

Postualte 5 (Euclid + Playfair)
For any straight line in the plane and for any point not lying on the line, there exists one and only one straight line that passes through the point and is parallel to the original line.

This way, for any point A in the plane, there exists a unique straight line that passes through A and is parallel to Oy. Similarly, there exists a unique straight line that passes through A and is parallel to Ox. The pair of the coordinates (x, y) of the corresponding intersection points on the lines forms the coordinates of point A in the plane.

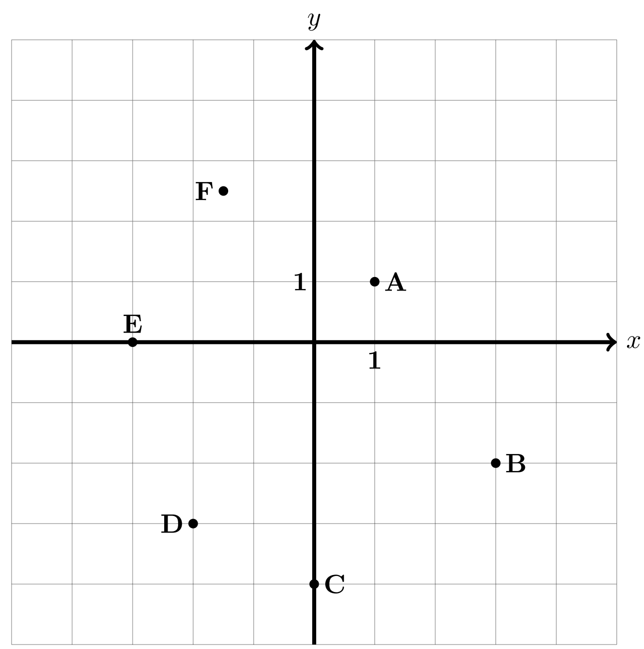


Problem 5 *What are the coordinates* (x_A, y_A) *of point* A*?*

$$A = (\quad , \quad)$$

Problem 6 *Draw point* $B = (7, 2)$ *on the picture above.*

It is more convenient to choose a coordinate system in the plane such that the angle between the Ox and Oy axes is right and the unit steps **i** and **j** have equal length.



Problem 7 *Write down the coordinates of the points on the picture above.*

$A = (\quad , \quad)$ $B = (\quad , \quad)$

$C = (\quad , \quad)$ $D = (\quad , \quad)$

$E = (\quad , \quad)$ $F = (\quad , \quad)$

Problem 8 *Draw the points* $G = (2, -3)$ *and* $H = (-4, -1)$ *on the picture above.*

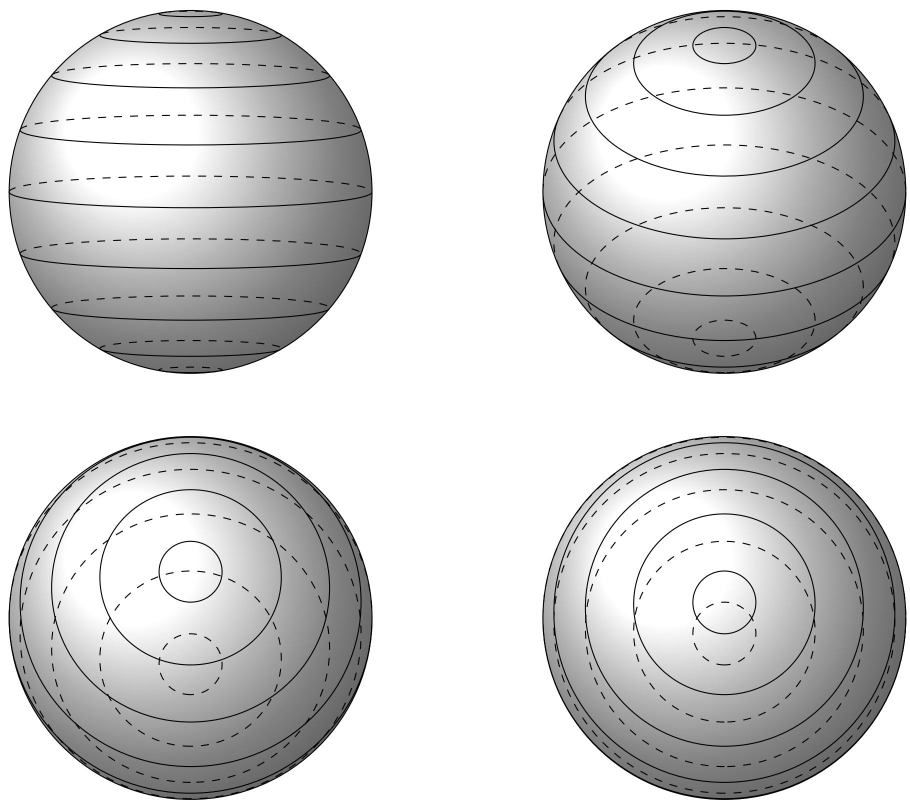
Coordinates on a sphere

There are a few ways to introduce coordinates on a sphere, each good for its own purpose. In this lesson, we will study the ones they use for navigating the globe.

The following observation should be familiar to anyone who has ever sliced a water-melon.

Fact 1 *A cross-section of a sphere by a plane is a circumference. The closer to the center of the sphere the cutting plane passes, the bigger is the radius of the cross-section.*

If you do not believe the above observation, please take a look at the following pictures showing parallel slices of a sphere at various angles.

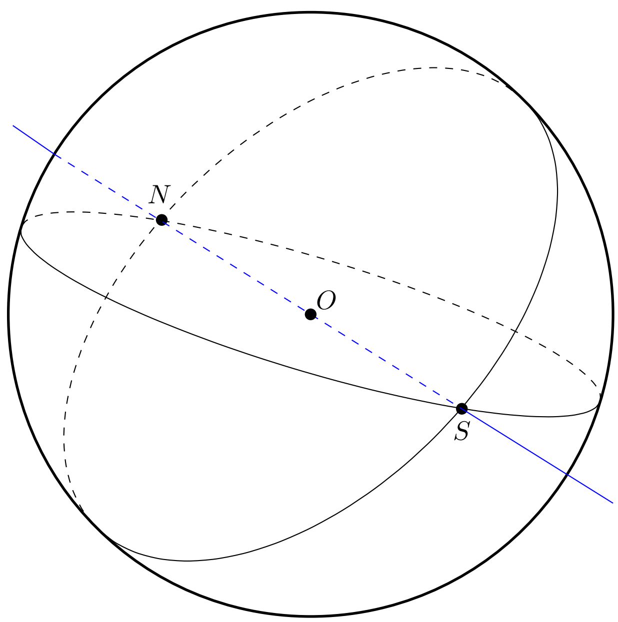


Note that as we move the cutting plane through the sphere from a pole to the equator, the radius of the cross-section circumference grows. It reaches its maximum when the plane passes through the center of the sphere. At this moment, the radius of the circumference equals to that of the sphere. Then the radius begins to subside.

Definition 1 *A great circle is a cross-section of a sphere by a plane passing through the sphere's center.*

In other words, a great circle is a circumference of the biggest radius we can draw on the sphere, that equal to the radius of the sphere itself.

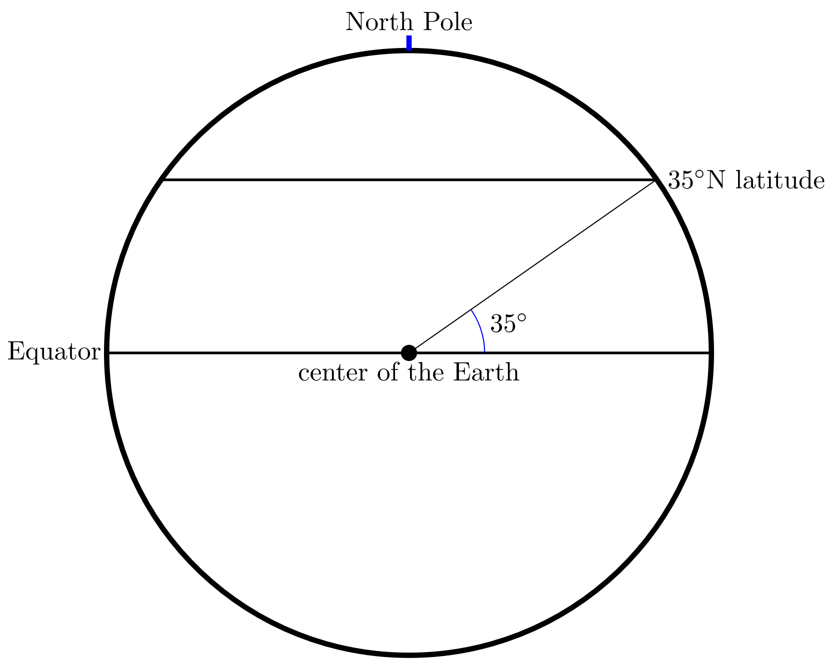
Halves of great circles connecting the poles on the globe are called the *longitude lines* or *meridians*. The meridians are marked with degrees, from 0° to 180°, East and West of the zero (a.k.a. prime) meridian, the one passing through the town of Greenwich in England. The angles marking meridians are the angles between the planes of the Greenwich meridian and the meridian in consideration. This is the reason why they use the 0° through 180° range East and West of Greenwich and not the uniform 0° through 360° range.



The meridians drawn on a model globe are called the grid meridians.

Problem 9 *The grid meridians come in* 15° *increments,* 0°, $15^\circ E$, $30^\circ E$, $45^\circ E$, *... Why?*

A *latitude* is the angle between the direction to the center of the Earth and the equatorial plane.



Latitude lines or *parallels* are the circles cut by the planes perpendicular to the Earth's axis of rotation on the surface of the globe. There is only one great circle among them. It is called the Equator.

Problem 10 *You are standing on the North Pole. You want to go West. Which way should you go?*

Problem 11 *What city has the following coordinates on the globe,* $(60°N, 30°E)$*? What country is it in?*

Problem 12 *Use the grid lines on the model globe to estimate the coordinates of the following cities.*

Los Angeles $= (\quad\quad , \quad\quad)$

New York City $= (\quad\quad , \quad\quad)$

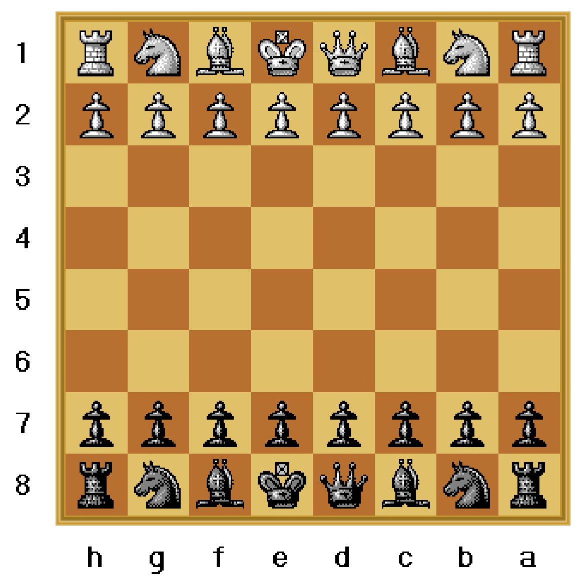
Washington, DC $= (\quad\quad , \quad\quad)$

Auckland, New Zealand $= (\quad\quad , \quad\quad)$

Coordinates can be used to mark not only points, but other objects as well. For example, they use the following coordinate system to mark squares on a chess board.

Problem 13 *The White open the game with the move e4. What piece do they move?*

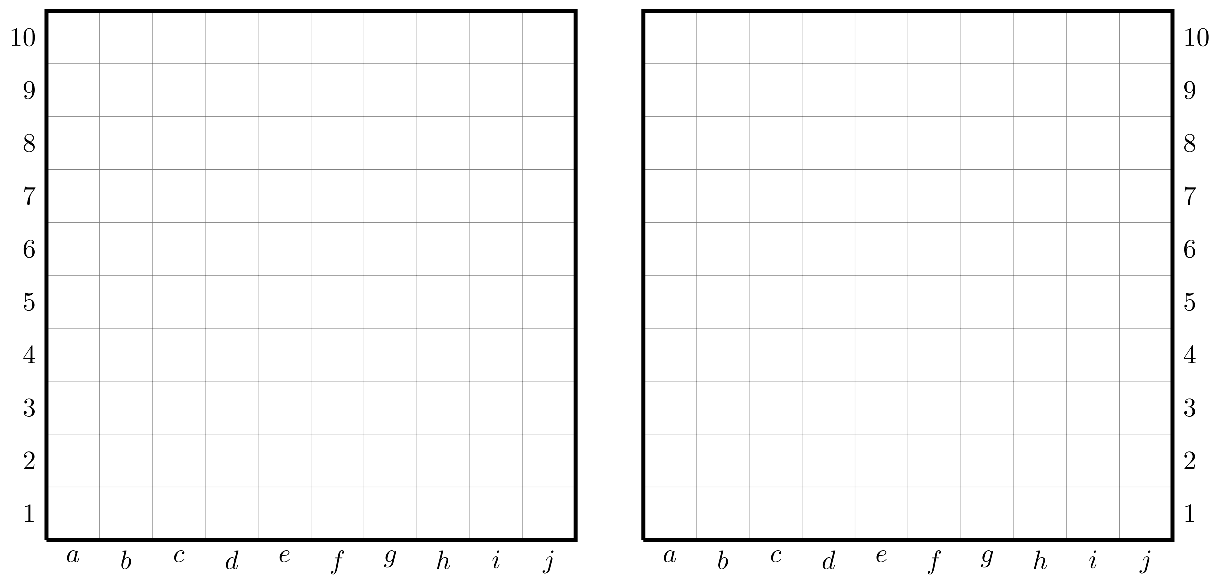
Mark the square they move it to on the picture below.

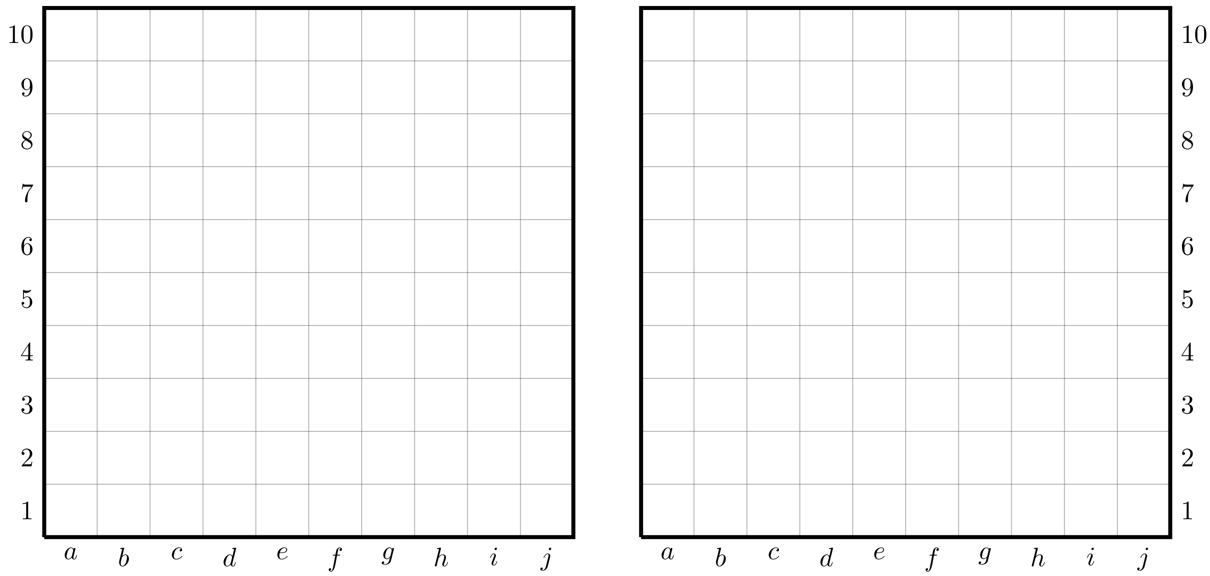


Write down you first move for the Black.

A similar coordinate system is used in the following popular game.

Battleships




10
9
8
7
6
5
4
3
2
1
a b c d e f g h i j
a b c d e f g h i j