Logarithms Continuation

Kason Ancelin

April 23, 2023

Exercise 1. *Solve the following equation.*

$4^{\sqrt{x+1}} = 64 * 2^{\sqrt{x+1}}$

Exercise 2. *Find the following values.*

$\log_{\sqrt{3}} 81$

$\log_6 3 + \log_6 2$

$\log_2 2\sqrt{2}$

$2 \log_4 2$

Exercise 3. *How is* $\log_b \sqrt{x}$ *related to* $\log_b x$*? How is* $\log_b \sqrt[n]{x}$ *related to* $\log_b x$*? (Hint: Recall one of the properties of the logarithm we proved in the last packet.)*

Exercise 4. *Estimate the number of decimal digits needed to write* 2^{100}*. (Hint: Recall* $\log_{10}$*)*

Exercise 5. *Given* $\log_{27} a = b$*, find* $\log_{\sqrt{3}} a^6$*.*

Exercise 6. *How is* $\log_b a$ *related to* $\log_a b$*?*

Exercise 7. *Use the fact shown in the previous exercise to find what* x^9 *equals in terms of* b *given the following equality* $\sqrt{log_b x} + \sqrt{\log_x b} = \frac{10}{3}$.

Carbon Dating

^{14}C is a radioactive isotope of carbon with *half-life time* $t_{1/2} = 5,700$ years. This means that if the amount of ^{14}C you have right now is $A(0) = 10$ grams, then 5,700 years later the amount left will be a half of that, $A(5,700) = 5$ grams.

Exercise 8.

- *What amount of the isotope would be left* $2 \times 5,700 = 11,400$ *years from now?*

- $3 \times 5,700 = 17,100$ *years from now?*

- $4 \times 5,700 = 22,800$ *years from now?*

- $n \times 5,700$ *years from now?*

Exercise 9. *If the original amount of* ^{14}C *was* $A(0)$*, how much would be left* T *years from now? The following formula should help.*

$$T = \frac{T}{t_{1/2}} \times t_{1/2} \tag{1}$$

^{14}C is produced by cosmic rays bombarding nitrogen atoms in the Earth's atmosphere. The resulting radioactive carbon combines with atmospheric oxygen to form radioactive carbon dioxide gas CO_2. The latter is digested by plants in the process of photosynthesis. It is passed further down the food chain from plants to plant-eating animals to carnivores. A living plant, or animal, has a stable level of ^{14}C in its body. Once the plant, or animal, dies, it stops exchanging the isotope with the environment. The level of ^{14}C starts decreasing according to the equation you have found in Problem 9. The idea to use the process of the ^{14}C decay for dating biological samples belongs to Willard Libby who received the Nobel Prize for his work in 1960.

Exercise 10. *A petrified redwood log has 1% of the original* ^{14}C *amount. How old is the sample?*

More Algebraic Log Problems

Exercise 11. *Solve:* $\log_{1+x}(2x^3 + 2x^2 - 3x + 1) = 3$

Exercise 12. *Solve:* $\log_3 (x-3)^2 + \log_3 |x-3| = 3$

Exercise 13. *Find the value of* $-\log_3 (\log_3 \sqrt[3]{\sqrt[3]{3}})$

Exercise 14. *Solve:* $\sqrt{x^{\log\sqrt{x}}} = 10$ *(recall that the* $\log$ *will be base 10 here by convention of this packet).*

A natural question to ask is how the size of the logarithms of different bases compare. For instance, how does $\log_2 x$ compare to $\log_{10} x$ for some fixed $x > 0$. A quick and simple calculation shows:

$$\log_2 x = \log_2 (10^{\log_{10} x}) = \log_{10} x * \log_2 10 \tag{2}$$

where the middle equality results from the $\log_{10}$ and 10^+ being inverses of each other and essentially 'cancelling out' and the rightmost equality using one of the properties of the logarithms proven on the previous packet (exponent in the argument of the logarithm can be pulled in front of the logarithm function as a constant). Here, we see for each $x > 0$ that $\log_2 x$ is $\log_2 10$ times larger than $\log_{10} x$. Quickly state and prove the generalization of this above calculation for arbitrary logarithm bases $a, b > 0$.

Exercise 15. *Use the statement shown above to show that if* $a = b^{\alpha}$ *for some real number* α*, then* $\log_b x = \alpha \log_a x$*. This fact may be quite useful for some of the upcoming problems.*

Exercise 16. *Solve:* $\log_x 9 + \log_{x^3} 729 = 10$

Exercise 17. *Solve:* $\sqrt{\log_2{(2x^2)}\log_4{(16x)}} = \log_4 x^3$