ORMC ARML Contest

March 12, 2023

1 Individual Problems

Rules: 2 problems at a time, 10 minutes for each pair of problems.

1. (ARML 2009 #2) Define a reverse prime to be a positive integer N such that when the digits of N are reversed, the resulting number is a prime. For example, the numbers 5, 6, and 110 are all reverse primes. Compute the largest two-digit integer N such that the numbers N, $4 \cdot N$, and $5 \cdot N$ are all reverse primes.

2. (ARML 2009 #1) Let p be a prime number. If p years ago, the ages of three children formed a geometric sequence with a sum of p and a common ratio of 2, compute the sum of the children's current ages.

3. (ARML 2009 #3) Some students in a gym class are wearing blue jerseys, and the rest are wearing red jerseys. There are exactly 25 ways to pick a team of three players that includes at least one player wearing each color. Compute the number of students in the class.

4. (ARML 2009 #5) Compute all real values of x such that $\log_2(\log_2(x)) = \log_4(\log_4(x))$.

5. (ARML 2011 #1) Compute the 2011th smallest positive integer N that gains an extra digit when doubled.

6. (ARML 2010 #1) Compute the number of positive integers less than 25 that cannot be written as the difference of two squares of integers.

7. (ARML 2011 #2) In triangle ABC, C is a right angle and M is on $\overline{AC}$. A circle with radius r is centered at M, is tangent to $\overline{AB}$, and is tangent to $\overline{BC}$ at C. If $AC = 5$ and $BC = 12$, compute r.

8. (ARML 2011 #3) The product of the first five terms of a geometric progression is 32. If the fourth term is 17, compute the second term.

9. (ARML 2011#5) A bag contains 20 lavendar marbles, 12 emerald marbles, and some number of orange marbles. If the probability of drawing an orange marble in one try is $\frac{1}{y}$, compute the sum of all possible integer values of y.

10. (ARML 2014 #5) The sequence of words $\{a_n\}$ is defined as follows: $a_1 = X$, $a_2 = O$, and for $n \geq 3$, a_n is a_{n-1} followed by the reverse of a_{n-2}. For example, $a_3 = OX$, $a_4 = OXO$, $a_5 = OXOXO$, and $a_6 = OXOXOOXO$. Compute the number of palindromes in the first 1000 terms of this sequence.

2 Relay Problems

2.1 Problems

ARML 2009 Relay 1:

1. A rectangular box has dimensions $8 \times 10 \times 12$. Compute the fraction of the box's volume that is *not* within 1 unit of the box's faces.

2. Let $T = TNYWR$. Compute the largest real solution x to $(\log x)^2 - \log \sqrt{x} = T$.

3. Let $T = TNYWR$. Kay has $T + 1$ different colors of fingernail polish. Compute the number of ways that Kay can paint the five fingernails on her left hand by using at least three colors and such that no two consecutive fingernails have the same color.

2.2 Official Rules/Instructions

Some students were curious what the official rules were, so here you go:

1. Each team has been broken down into groups of 3. Each of the 3 people will receive a different problem. When the first person solves their problem, they writes the answer on a small piece of paper and passes it back/to the right. The second person needs that number to solve their problem (the number is referred to as "the number you will receive," or TNYWR). When the second person gets an answer, they pass that number back/to the right. When the third person gets their answer, they write it on an official answer slip, and that slip will get handed in.

2. The second and third people can usually do most of their work before getting a number from the person ahead of them. Nothing may be passed except a number– NO COMMENTS, NO OTHER INFORMATION. You should double underline the answer you pass in case it accidentally gets turned upside down and looks like a different number. You may continue to pass answers, even if they are the same as before. NOTHING MAY BE PASSED FORWARD/TO THE LEFT IF YOU RECEIVE AN ANSWER THAT IS IMPOSSIBLE FOR YOU TO USE, JUST WAIT AND HOPE THAT A DIFFERENT ANSWER WILL GET PASSED TO YOU!

3. The third person may hand in an answer only at the 3 minute mark and at the 6 minute mark. If the answer handed in at 3 minutes is correct,

and no answer is handed in later, it gets maximum points. If any answer is handed in at 6 minutes, the previous answer is discarded by the proctor (even if the number is the same as before!), and you can only get the lower number of points, provided the answer is correct. OBVIOUSLY, THE THIRD PERSON SHOULD NOT HAND IN THE SAME ANSWER A SECOND TIME!

4. During the Relay round, only the answer to the question in the student's possession can be passed on to the next person on the team. This answer may be underlined to show orientation but that is the only mark that can be on the paper.

5. The method of passing of answer(s) should, in no way, indicate anything besides the answer to the question. No comments or any other information may be communicated at any time during the relay round. In the past, we have found that ARML participants have devised methods to communicate with members of their team. These practices are violations of the rules of the competition. Such practices include, but are not limited to: passing back multiple pieces of paper with the same answer at the same time in an effort to communicate that the participant is very certain of their computation, or a participant audibly crumpling and/or throwing a piece of paper on the floor to communicate to another participant that their submitted answer cannot be used in subsequent problems. Participants may underline answers so that they will be read correctly, to distinguish between a 6 and a 9, for example. In the spirit of fair and honest competition, and so that all individual teams are competing on the same playing field, we hope that all participants will abide by the rules as they were originally intended. Relay teams that are found to be communicating more than the answer to the problem will possibly face disqualification. Specific penalties will be decided by the ARML officials.