Tuesday, FEBRUARY 11, 2003

Contest A

The MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

Presented by the Akamai Foundation

AMC 10

4th Annual American Mathematics Contest 10

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score in the top 1% on this AMC 10 will be invited to take the 21st annual American Invitational Mathematics Examination (AIME) on Tuesday, March 25, 2003 or on Tuesday, April 8, 2003. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 2003 **(E)** 4006

2. Members of the Rockham Soccer League buy socks and T–shirts. Socks cost \$4 per pair and each T–shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

   (A) 77 **(B)** 91 **(C)** 143 **(D)** 182 **(E)** 286

3. A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

   (A) 4.5 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

4. It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?

   (A) 3 **(B)** 3.125 **(C)** 3.5 **(D)** 4 **(E)** 4.5

5. Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of $(d-1)(e-1)$?

   (A) $-\frac{5}{2}$ **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 6

6. Define $x\heartsuit y$ to be $|x-y|$ for all real numbers x and y. Which of the following statements is **not** true?

   (A) $x\heartsuit y = y\heartsuit x$ for all x and y

   (B) $2(x\heartsuit y) = (2x)\heartsuit(2y)$ for all x and y **(C)** $x\heartsuit 0 = x$ for all x

   (D) $x\heartsuit x = 0$ for all x **(E)** $x\heartsuit y > 0$ if $x \neq y$

7. How many non-congruent triangles with perimeter 7 have integer side lengths?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. What is the probability that a randomly drawn positive factor of 60 is less than 7?

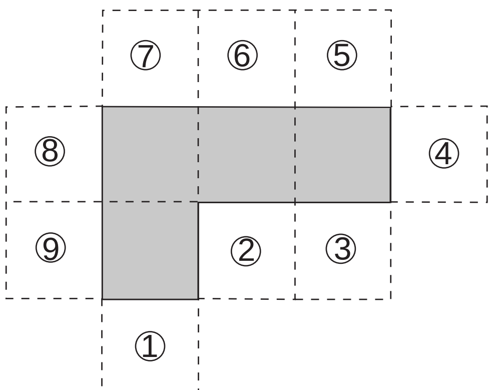
(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

9. Simplify

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}.$$

(A) $\sqrt{x}$ **(B)** $\sqrt[3]{x^2}$ **(C)** $\sqrt[27]{x^2}$ **(D)** $\sqrt[54]{x}$ **(E)** $\sqrt[81]{x^{80}}$

10. The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

11. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

12. A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{4}$

13. The sum of three numbers is 20. The first is 4 times the sum of the other two. The second is seven times the third. What is the product of all three?

(A) 28 **(B)** 40 **(C)** 100 **(D)** 400 **(E)** 800

14. Let n be the largest integer that is the product of exactly 3 distinct prime numbers, d, e and $10d+e$, where d and e are single digits. What is the sum of the digits of n?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 24

15. What is the probability that an integer in the set $\{1, 2, 3, \ldots, 100\}$ is divisible by 2 and not divisible by 3?

(A) $\frac{1}{6}$ **(B)** $\frac{33}{100}$ **(C)** $\frac{17}{50}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{18}{25}$

16. What is the units digit of 13^{2003}?

(A) 1 **(B)** 3 **(C)** 7 **(D)** 8 **(E)** 9

17. The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

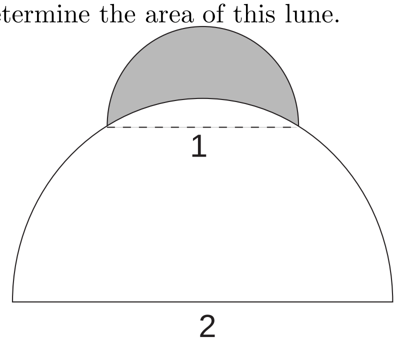
(A) $\frac{3\sqrt{2}}{\pi}$ **(B)** $\frac{3\sqrt{3}}{\pi}$ **(C)** $\sqrt{3}$ **(D)** $\frac{6}{\pi}$ **(E)** $\sqrt{3}\pi$

18. What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

(A) $-\frac{2004}{2003}$ **(B)** -1 **(C)** $\frac{2003}{2004}$ **(D)** 1 **(E)** $\frac{2004}{2003}$

19. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ **(D)** $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$

(E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

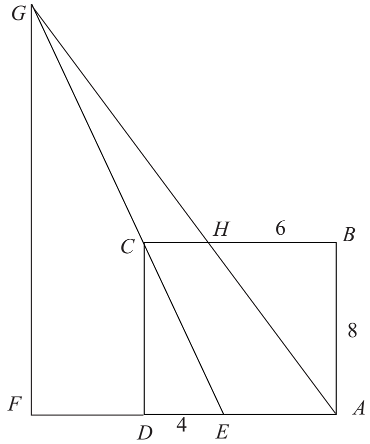
20. A base-10 three-digit number n is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of n are both three-digit numerals?

(A) 0.3 **(B)** 0.4 **(C)** 0.5 **(D)** 0.6 **(E)** 0.7

21. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

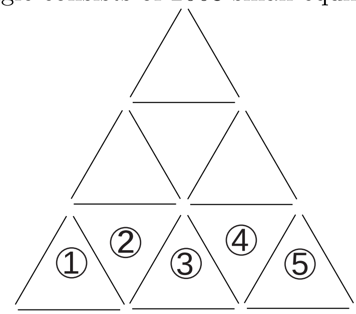
(A) 22 **(B)** 25 **(C)** 27 **(D)** 28 **(E)** 729

22. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on $\overline{BC}$ with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $\overline{GF} \perp \overline{AF}$. Find the length $\overline{GF}$.



(A) 16 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

23. A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles?



(A) 1,004,004 **(B)** 1,005,006 **(C)** 1,507,509 **(D)** 3,015,018 **(E)** 6,021,018

24. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

25. Let n be a 5-digit number, and let q and r be the quotient and remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

(A) 8180 **(B)** 8181 **(C)** 8182 **(D)** 9000 **(E)** 9090

WRITE TO US!

Correspondence about the problems and solutions for this ***AMC 10*** *should be addressed to:*

Prof. Douglas Faires, Department of Mathematics
Youngstown State University, Youngstown, OH 44555-0001
Phone:330-742-1805; Fax: 330-742-3170; email: faires@math.ysu.edu

Orders for any of the publications listed below should be addressed to:

Titu Andreescu, Director
American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu;

2003 AIME

The AIME will be held on Tuesday, March 25, 2003 with the alternate on April 8,2003. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of this AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN US FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD/AMERICAN EXPRESS accepted. Include card number, expiration date, cardholder name and address. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.

INTERNATIONAL ORDERS: Do **NOT** prepay. An invoice will be sent to you.

Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2003.

- AMC 10 2000-2003/AHSME (AMC 12) 1989-2003, $1 per copy per year.
- AIME 1989-2003, $2 per copy per year (2003 available after April)
- USA and International Math Olympiads, 1989-1999, $5 per copy per year; 2000, 2001 - $14.00 each
- National Summary of Results and Awards, 1989-2003, $10 per copy per year.
- Problem Book I, AHSMEs 1950-60, Problem Book II, AHSMEs 1961-65, $10/ea
- Problem Book III, AHSMEs 1966-72, Problem Book IV, AHSMEs 1973-82, $13/ea
- Problem Book V, AHSMEs and AIMEs 1983-88, $30/ea
- Problem Book VI, AHSMEs 1989-1994, $24/ea
- USA Mathematical Olympiad Book 1972-86, $18/ea
- International Mathematical Olympiad Book II, 1978-85, $20/ea
- World Olympiad Problems/Solutions 1995-96, 1996-97, 1997-98, $15/ea
- Mathematical Olympiads Problems & Solutions from around the World 1998-1999, 1999-2000, $25/ea
- The Arbelos, Volumes I-V, and a Special Geometry Issue, $8/ea

Shipping & Handling charges for Publication Orders:

Order Total	Add:	Order Total	Add:
$ 10.00 -- $ 30.00	$ 5	$ 40.01 -- $ 50.00	$ 9
$ 30.01 -- $ 40.00	$ 7	$ 50.01 -- $ 75.00	$12
		$ 75.01 -- up	$15

2003

AMC 10 - Contest A

DO NOT OPEN UNTIL
Tuesday, FEBRUARY 11, 2003

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 11.** Nothing is needed from inside this package until February 11.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

Sponsored by
The MATHEMATICAL ASSOCIATION OF AMERICA
The Akamai Foundation
University of Nebraska – Lincoln
Contributors
American Statistical Association
Casualty Actuarial Society
Society of Actuaries
National Council of Teachers of Mathematics
American Society of Pension Actuaries
American Mathematical Society
American Mathematical Association of Two Year Colleges
Pi Mu Epsilon
Consortium for Mathematics and its Applications
Mu Alpha Theta
National Association of Mathematicians
Kappa Mu Epsilon
School Science and Mathematics Association
Clay Mathematics Institute
Institute for Operations Research and the Management Sciences
Canada/USA Mathpath & Mathcamp