

## GRAPH III

MATH CIRCLE (INTERMEDIATE) 6/3/2012

1) Can a kingdom in which 5 road lead out of each city have exactly 124 roads?

2) Prove that there does not exist a graph with 5 vertices with degrees equal to 4, 4, 4, 4, and 2.

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Definition: A tree is a connected graph without cycles.

3) Prove that a graph in which any two vertices are connected by one and only one simple path is a tree.

4) Is there a tree with 11 vertices and 11 edges?

Challenge 1) Prove that in any tree having at least one edge there exists a vertex which is an endpoint of exactly one edge.

Challenge 2) prove that if an edge (excluding its ends) is deleted from a tree, then the resulting graph is not connected.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes

Warm up 1) In a certain kingdom, there are 50 cities, and five roads lead out of each city. How many roads are there altogether in this kingdom?

Warm up 2) There are 30 students in a class. Can it happen that 9 of them have 3 friends each (in the class), eleven have 4 friends each, and ten have 5 friends each?