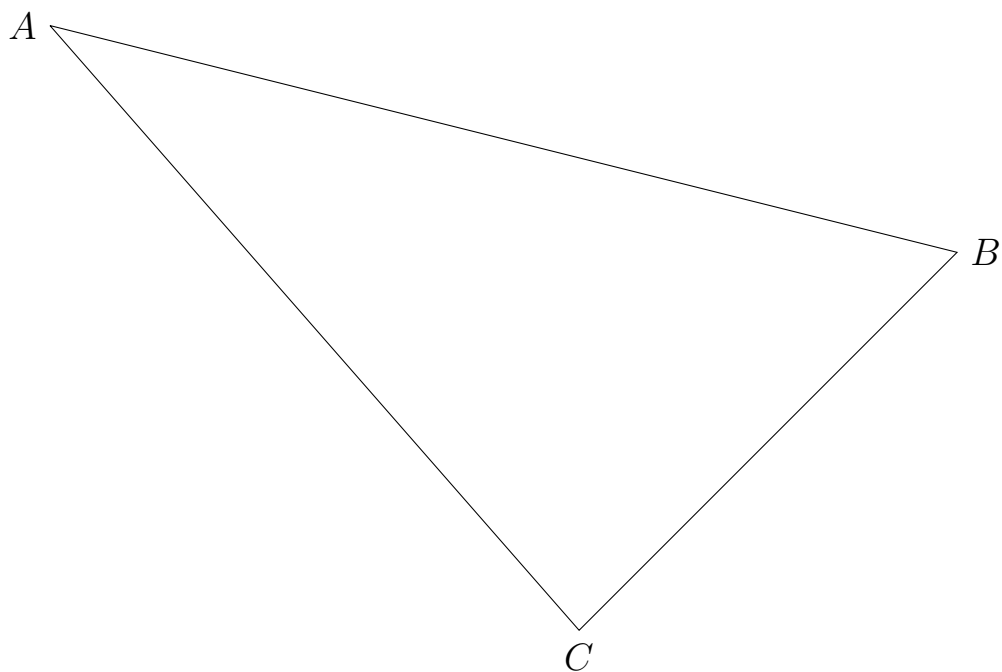


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**Problem 1** *Using a protractor, measure the angles  $\angle A$  and  $\angle B$  of the triangle below. Find the size of the angle  $\angle C$ , in degrees, without measuring it.*

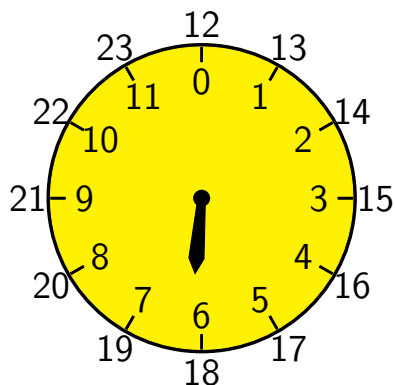


$$\angle C =$$

**Problem 2** *It takes a grandfather's clock 30 seconds to chime 6 o'clock. How much time would it take the clock to chime 12?*

### Clock Arithmetic or a Circle as a Number Line

One way to turn a circle into a number line is to divide it into twelve equal parts. In this case, one step is usually called one hour.



0 coincides with 12. The hour hand moves from 0 to 1, from 1 to 2, ... from 11 to 12 just as it would have on the straight number line. However, 12 equals 0 on this circle, so there it goes

again, from 1 to 2, and so on. We write down the fact that 12 equals 0 as

$$12 \equiv 0 \pmod{12} \tag{1}$$

and read it as *12 is congruent to 0 modulo 12*. The usual “=” sign is reserved for the straight number line; we use “ $\equiv$ ” on the circle instead. The *mod* 12 symbol tells us that the circle is divided into 12 equal parts, so 12 coincides with 0, 13 – with 1, 14 – with 2, and so on. Or in the new notations,

$$13 \equiv 1 \pmod{12}, 14 \equiv 2 \pmod{12}, \dots, 23 \equiv 11 \pmod{12},$$

$$24 \equiv 12 \equiv 0 \pmod{12}.$$

### **Problem 3**

$$21 \pmod{12} \equiv$$

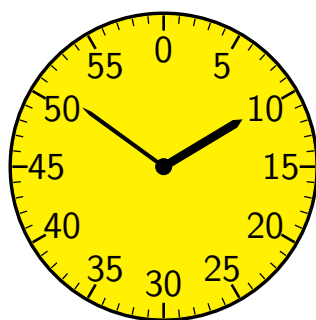
$$80 \pmod{12} \equiv$$

$$9 + 4 \equiv \qquad \pmod{12}$$

$$24 - 2 \equiv \qquad \pmod{12}$$

**Problem 4** *An experiment in a biological lab starts at 7:00 AM and runs for 80 hours. What time will it end?*

Another standard way to turn a circle into a number line is to divide it into 60 equal parts. Depending on the situation, the unit step is called either a minute or a second.



All the numbers living on this number line are considered modulo 60. In particular,  $60 \equiv 0 \pmod{60}$ . There are 60 minutes in an hour.

**Problem 5**

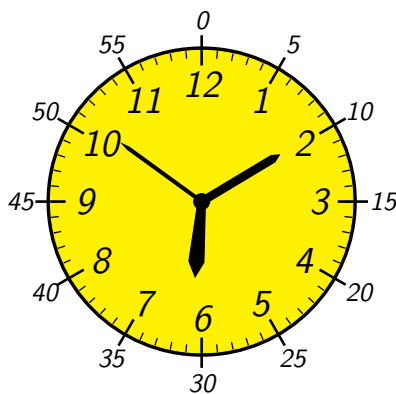
$$72 \pmod{60} \equiv$$

$$135 \pmod{60} \equiv$$

$$55 + 55 \equiv \quad \pmod{60}$$

$$240 - 59 \equiv \quad \pmod{60}$$

**Problem 6** *What is the time, in hours, minutes, and seconds, on the clock below?*



There are 24 hours in a day, so one more standard way to turn a cricle into a number line is to divide it into 24 equal parts. The US military use the 24-hour clock. The following is a photograph of the 24-hour clock from the USS (United States Ship) *Mullinnix*, the last “all gun” US Navy destroyer in the Pacific, decommissioned in 1982.<sup>1</sup>



USS *Mullinnix* 24-hour clock.<sup>2</sup>

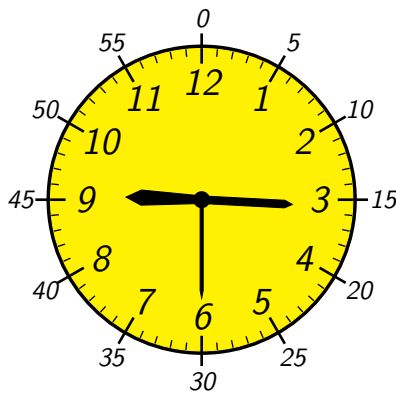
<sup>1</sup>See its homepage at <http://www.ussmullinnix.org/>

<sup>2</sup>Downloaded from <http://www.ussmullinnix.org/MuxMemorabilia.html>

Since  $60 \div 24$  is not a whole number, we can't use the same marks on the face of a 24-hour clock for minutes and hours (to better see this, please find the minute and hour marks on the face of the USS *Mullinnix* clock).  $60 \div 12 = 5$ , so this inconvenience doesn't exist for the clocks and watches we are used to. On the other hand, to disambiguate between, say, 1 o'clock night time and 1 o'clock afternoon, we have to use the A.M./P.M. notation not needed in the military. In their language, 1 o'clock P.M. is 13:00 (thirteen hundred) hours, plain and simple.

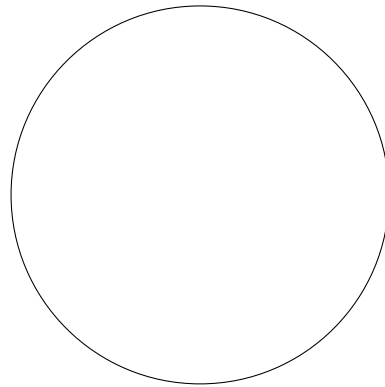
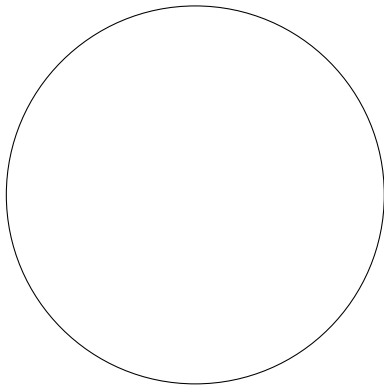
**Problem 7** *What time does the USS Mullinnix clock show?*

**Problem 8** *What is the time on the clock below?*



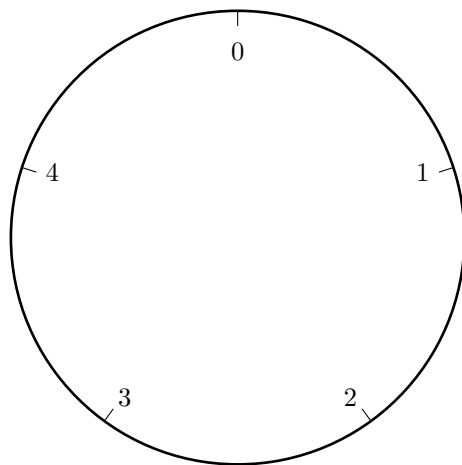
*Suppose that this is the time P.M. How would the military call it?*

**Problem 9** *On the left, draw the civilian clock showing 1:45. On the right, draw the military clock showing the same time P.M.*





In the following problems, we will consider the *mod* 5 arithmetic, that of a circle divided into five equal parts.



**Problem 10**

$$7 \pmod{5} \equiv$$

$$100 \pmod{5} \equiv$$

$$-1 \pmod{5} \equiv$$

$$1 - 4 \equiv \quad \pmod{5}$$

$$3 + 3 \equiv \quad (\textit{mod } 5)$$

$$2 \times 3 \equiv \quad (\textit{mod } 5)$$

$$1 \div 2 \equiv \quad (\textit{mod } 5)$$

$$1 \div 3 \equiv \quad (\textit{mod } 5)$$

$$4 \times 4 \equiv \quad (\textit{mod } 5)$$

$$1 \div 4 \equiv \quad (\textit{mod } 5)$$