Axiom Reference Sheet

It can be really tough to remember all of the algebraic axioms at first. To help you out, here is a reference sheet listing them all! We also include a table showing which types of numbers on the real line satisfy which axioms.

Let S be a set of objects. Suppose we can add and multiply together the objects in S, meaning there are binary operations $+$ and $\cdot$ on S. Then, we have the following definitions of algebraic axioms on S (we put formal and casual versions to help you remember).

Axiom 1 (Closure of addition). *We say that S is closed under addition if the following is true.*

- *Formally: $x + y$ is in S whenever x, y are in S.*
- *Casually: addition keeps you inside of S.*

Axiom 2 (Closure of multiplication). *We say that S is closed under multiplication if the following is true.*

- *Formally: $x \cdot y$ is in S whenever x, y are in S.*
- *Casually: multiplication keeps you inside of S.*

Axiom 3 (Associativity of addition). *We say that S satisfies the associativity of addition if the following is true.*

- *Formally: $x + (y + z) = (x + y) + z$ for objects x, y, z in S.*
- *Casually: we can ignore parentheses when adding.*

Axiom 4 (Associativity of multiplication). *We say that S satisfies the associativity of multiplication if the following is true.*

- *Formally: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for objects x, y, z in S.*
- *Casually: we can ignore parentheses when multiplying.*

Axiom 5 (Commutativity of addition)**.** *We say that S satisfies the commutativity of addition if the following is true.*

- *Formally: $x + y = y + x$ for objects x, y in S.*
- *Casually: we can ignore order when adding.*

Axiom 6 (Commutativity of multiplication)**.** *We say that S satisfies the commutativity of multiplication if the following is true.*

- *Formally: $x \cdot y = y \cdot x$ for objects x, y in S.*
- *Casually: we can ignore order when multiplying.*

Axiom 7 (Identity of addition)**.** *We say that S has an additive identity a if the following is true.*

- *Formally: There is an object a in S such that $a + x = x + a = x$ for all objects x in S.*
- *Casually: There's an object that does nothing when adding with it.*

Axiom 8 (Identity of multiplication)**.** *We say that S has a multiplicative identity m if the following is true.*

- *Formally: There is an object m in S such that $m \cdot x = x \cdot m = x$ for all objects x in S.*
- *Casually: There's an object that does nothing when multiplying by it.*

Axiom 9 (Inverse of addition)**.** *Suppose S has an additive identity a. We say that S satisfies the additive inverse axiom if the following is true.*

- *Formally: For an object x in S, there is a corresponding object y in S such that $y + x = x + y = a$. We call y the additive inverse of x.*
- *Casually: There's always an opposite object when adding.*

Axiom 10 (Inverse of multiplication)**.** *Suppose S has a multiplicative identity m. We say that S satisfies the multiplicative inverse axiom if the following is true.*

- *Formally: For an object x in S (not equal to the additive identity a if it exists in S), there is a corresponding object y in S such that $y \cdot x = x \cdot y = m$. We call y the multiplicative inverse of x.*
- *Casually: There's always an opposite object when multiplying (except for the additive identity a if it exists in S).*

Axiom 11 (Left distributivity)**.** *We say that S satisfies left distributivity if the following is true.*

- *Formally: $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ for objects x, y, z in S.*
- *Casually: Multiplication jumps over addition on the left.*

Axiom 12 (Right distributivity)**.** *We say that S satisfies right distributivity if the following is true.*

- *Formally: $(y + z) \cdot x = (y \cdot x) + (z \cdot x)$ for objects x, y, z in S.*
- *Casually: Multiplication jumps over addition on the right.*

Recall the types of numbers on the real line from the second handout (What is a Number Part II): naturals $\mathbb{N}$, wholes $\mathbb{W}$, integers $\mathbb{Z}$, rationals $\mathbb{Q}$, reals $\mathbb{R}$. We also had an extra credit discussion about the irrationals $\mathbb{P}$ which are defined to be all the real numbers that are not rational. Below is a chart of which number type satisfies the above axioms. A checkmark ✓ means the axiom is satisfied while an X means it is not.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{P}$
+ closure	✓	✓	✓	✓	✓	X
+ associativity	✓	✓	✓	✓	✓	✓
+ commutativity	✓	✓	✓	✓	✓	✓
+ identity	X	✓	✓	✓	✓	X
+ inverse	X	X	✓	✓	✓	✓
· closure	✓	✓	✓	✓	✓	X
· associativity	✓	✓	✓	✓	✓	✓
· commutativity	✓	✓	✓	✓	✓	✓
· identity	✓	✓	✓	✓	✓	X
· inverse	X	X	X	✓	✓	✓
Left distributivity	✓	✓	✓	✓	✓	✓
Right distributivity	✓	✓	✓	✓	✓	✓