What Is a Number? (Part I)

Our goal this quarter is to understand the card game *SET* from a mathematical point of view. In particular, we hope to model *SET* in terms of some math that will let us easily beat the game because that's what fun is all about. But what does it mean to "model" something with math and how would we go about doing so? We will try to understand these questions over the next few lessons through the lens of "Abstract Algebra". Of course, the best place to start is always at the beginning; so, let's brainstorm about the very first and oldest mathematical model: numbers!

What really *is* a number in the first place?

The following four problems are asking for your thoughts, ideas, and opinions. There are no "most correct" answers. So, take your time and write down everything that comes to mind! Feel free to work with your classmates.

Problem 1. *What do you think a number is?*

Problem 2. *Write down all of the types of numbers you can think of, along with a few examples for each type. To get you started,* -1 *is an integer.*

Problem 3. *For each type of number you came up with in Problem 2, explain how people use that type of number in the real world. For example, the natural/whole numbers are used to count and order objects.*

Problem 4. *How different are the real-world uses of numbers that you gave in Problem 3? Do these uses make some types of numbers more "real" than others, in your opinion? In particular, are there any types of numbers that you think are not actually "real" and why?*

What do numbers *do*?

Now, let's shift our focus to what we can do with numbers. This will help us better understand how to use numbers to model the real world.

Problem 5. *What are some simple ways of combining two numbers into one? We call these (binary) operations.*

Problem 6. *Do each of the operations that you listed in Problem 5 work on all types of numbers you listed in Problem 2? When you apply these operations, does the type of number change?*

Problem 7. *Write down all of the rules/laws you can think of that each of the operations (that you listed in Problem 5) follow. To get you started, addition is commutative, meaning $a + b = b + a$ for any numbers a, b of any type.*

Problem 8. *For each operation (that you listed in Problem 5), is there a number that doesn't change other numbers when using that operation?*