MATH CIRCLE - FALL 2022 - EXAM PART 2

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Name:	

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Problem 1: In the TV show Futurama, Professor Farnsworth creates a machine that allows people to switch bodies. However, the machine has a fundamental flaw, two people cannot swap bodies twice. This means that, if X and Y swap bodies, we can't swap Y back to X immediately. However, maybe, we could use a third person to swap bodies back.

Show that, regardless of how many swaps have been made, they can still all be restored to their original bodies using extra people.

Extra question: What is the least number of extra people that is needed?

Solution 1:

Problem 2: Consider the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 15 & 14 & [\ ] \end{bmatrix}$$

The [ ] is an empty square. You can slide any adjacent number into the empty square, and the position of such number will become empty.

Is it possible to put the matrix in numerical order with the lower left corner empty?

Is it possible to put the matrix in numerical order with the lower right corner empty?

Solution 2:

Problem 3: A real number is said to be *constructible* if we can draw a line segment of length $|r|$ given a unit line segment, a compass, and a straightedge.[1]

Show that the number $\sqrt{13+\sqrt{5}}$ is constructible.

Solution 3:

[1]The compass can't be used to remember distances and the straightedge does not have measures on it

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