

**MATH CIRCLE - FALL 2022 - EXAM PART 1**

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Name:	

Problem 1	/10
Problem 2	/10
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Total	/30

**Problem 1:** Consider a chessboard. We cut off two opposite corner pieces of the chessboard and call this a *truncated board*. A *domino* is a  $2 \times 1$  rectangular piece. A *Tetris* is a T-shaped piece that is constructed with 4 blocks.

- a) Can you fill the chess board using dominos?
- b) Can you fill the truncated board using dominos?
- c) Can you fill the chess board using Tetris?
- d) Can you fill the truncated board using Tetris?

**Solution 1:**



**Problem 2:** The points in the plane are colored in three colors red, green, and blue. Show that there is a rectangle whose vertices have the same color.

**Solution 2:**



**Problem 3:** We introduce the game *sprouts*. Two players play the game. We start with a few spots drawn on a sheet, these spots are called *sprouts*. Players take turns. Each turn, the player adds a line (called *root*) between two sprouts and add a new sprout somewhere along the line. The players are constrained to the following rules:

- (1) the lines may be straight or curved, but must not touch or cross itself or any other line,
- (2) the new sprout cannot be placed on top of one of the endpoints of the new root,
- (3) no sprout may have more than three roots attached to it. A root that starts and ends at the same sprouts counts as two roots from it.

Show that the sprouts game always terminates independently of the number of initial sprouts.

**Solution 3:**



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