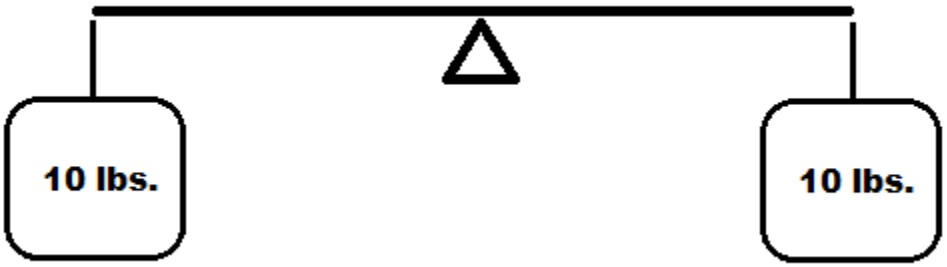
A LEVER PLAYING FIELD

MATH CIRCLE (BEGINNERS) 05/20/2012

You have a scale which has a rigid bar from which weights can be hung. The point at the center on which the scale balances is called the fulcrum. Use your intuition and experience to answer the following:



(1) If you have **equal** weights at **equal** distances from the fulcrum, the scale is:

(a) tilted down on the left

(b) balanced

(c) tilted down on the right

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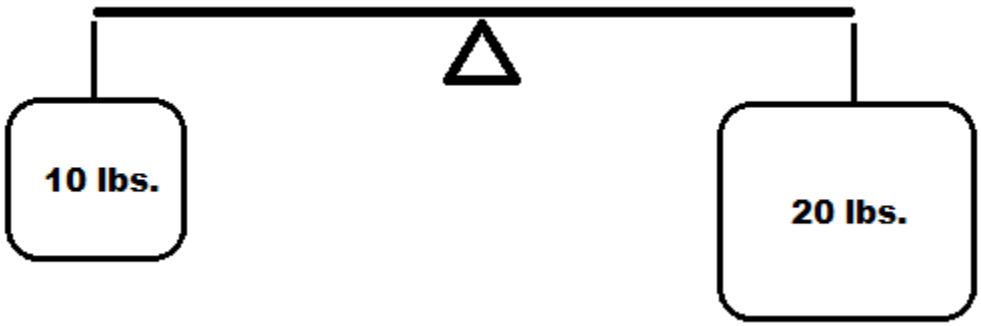

(2) If you have **unequal** weights at **equal** distances from the fulcrum, the scale is:

(a) tilted down on the side with the lighter weight

(b) balanced

(c) tilted down on the side with the heavier weight

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(3) If you have **equal** weights at **unequal** distances from the fulcrum, the scale is

(a) tilted down on the side with the weight closer to the fulcrum

(b) balanced

(c) tilted down on the side with the weight further from the fulcrum

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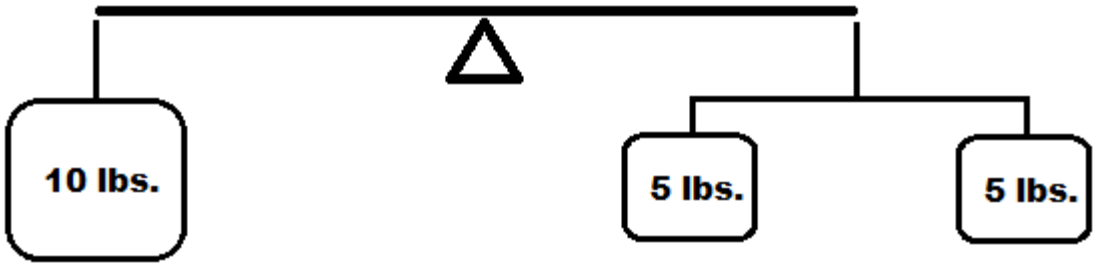


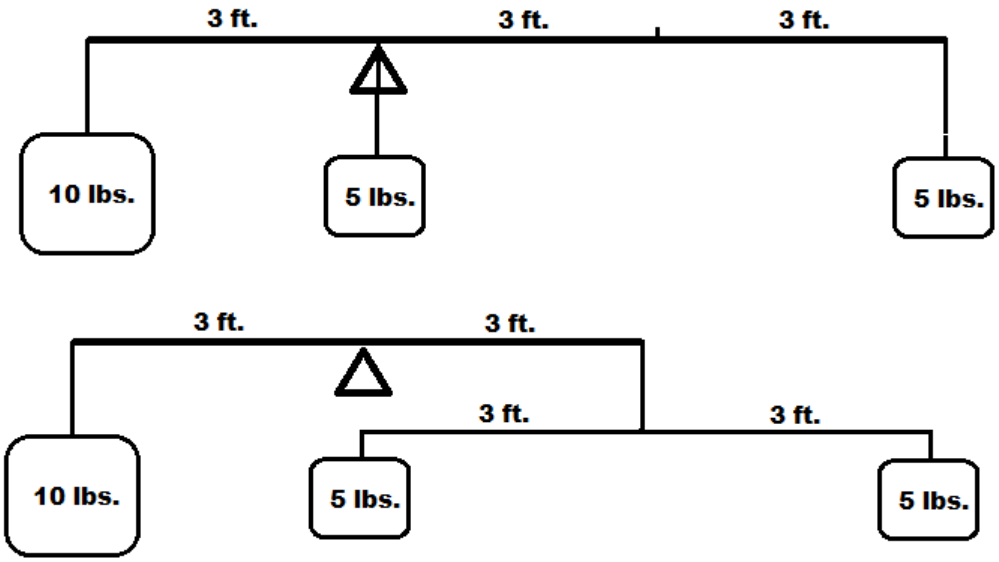
(4) The 5-lb. weights above are at equal distance from the cord their bar is hanging from (and that cord and the 10-lb. weight are at equal distances from the fulcrum). The scale will

(a) tilt down on the left

(b) balance

(c) tilt down on the right

We can actually hang weights directly from the top bar and it doesn't make a difference—so the following two pictures are equivalent:



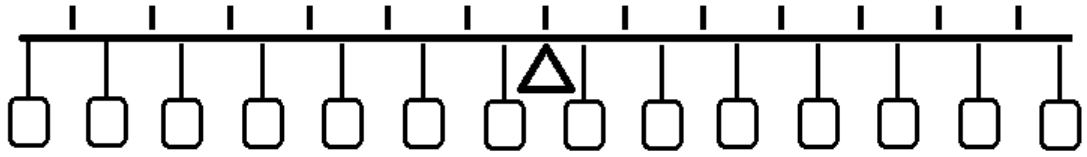
(5) In these configurations, the balance

(a) tilts down on the left

(b) balances

(c) tilts down on the right

(6) What will happen if we remove the 5-lb. weight directly beneath the fulcrum?



(7) The balance above has 14 small, 1-lb. weights suspended evenly across the top bar. (The marks on top are 1-foot apart—they are just there to help you measure distances.) Suppose you wanted to replace the **first 8** of them with a single large weight.

(a) How much should it weigh?

(b) Where should you place it? (How far from the fulcrum, on which side?)

Draw a picture of the new version, with the first 8 replaced. Label the distance and weight of the new weight:

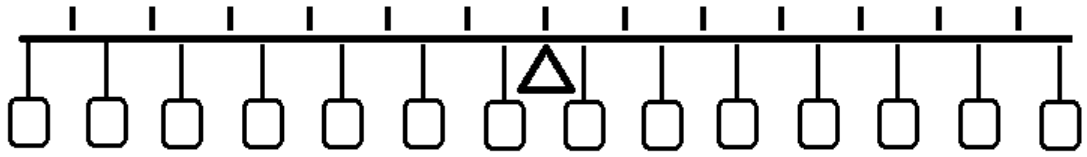
(8) Now that you've replaced the first 8 small weights with one big one, you'd like to replace the **last 6** of them with another single weight.

(a) How much should it weigh?

(b) Where should you place it?

Draw a picture of the new version, with both first 8 and last 6 replaced. Label the distances and weights of the two new weights:

Here's the 14 small weights again:



(9) Now I want to replace the **first 4** small weights with one weight, and the **last 8** small weights with a second large weight.

Draw a picture of the new version, labelling the distances and weights of the two new weights:

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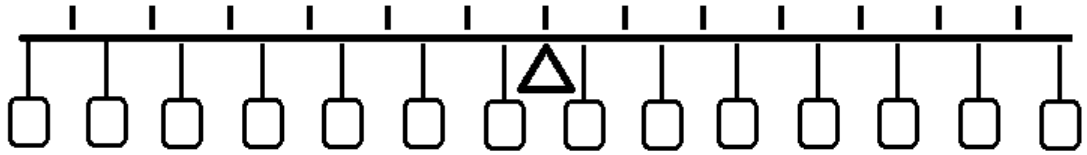

(10) Now replace the first 13 weights with a single weight, and leave the very last weight there.

Draw a picture of the new version, labelling the distances and weights of the two weights:

(11) In questions 7–10, would it have been possible to give different answers than the ones you did?

The Law of the Lever, first proved by the Greek mathematician Archimedes, describes the situation in which a scale/balance/lever will be balanced.

Consider a lever that has a weight of W_1, at a distance D_1 to the left of the fulcrum, and a weight of W_2 at distance D_2 to the right of the fulcrum (as pictured). Then the Law of the Lever states that the two weights will be balanced if, and only if

$$W_1 D_1 = W_2 D_2.$$

More generally, if there are multiple weights on each side, then the *sum* of the weight$\times$distance values on the left side, must equal the sum of the weight$\times$distance values on the right side.

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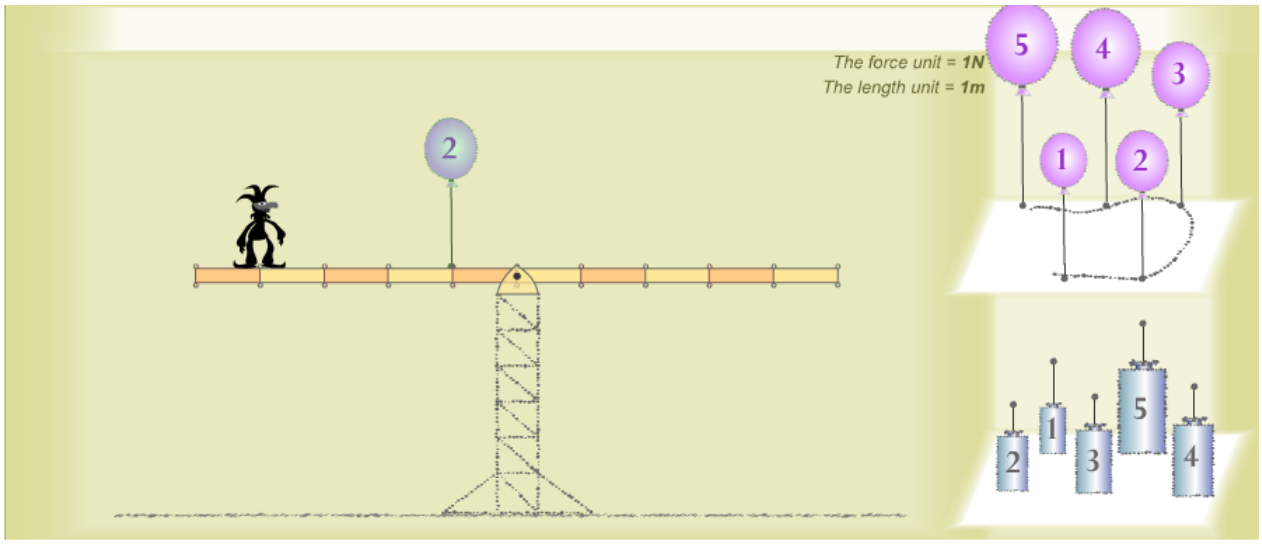
In the following pictures, your goal is to balance the beam. The jester weighs 1 unit of downward force, the weights have 1, 2, 3, 4, or 5 units of downward force, and the balloons weigh 1, 2, 3, 4, or 5 but they exert force *upward*.

For each picture, circle each balloon and weight which could be used to balance the beam if **only that weight/balloon** were attached to a single location on the bar that has a hook (note: hooks appear at distances of 1, 2, 3, 4, and 5 from the center on the left and right). Then draw at least one way that it can be done, using one of the weights/balloons you circled.



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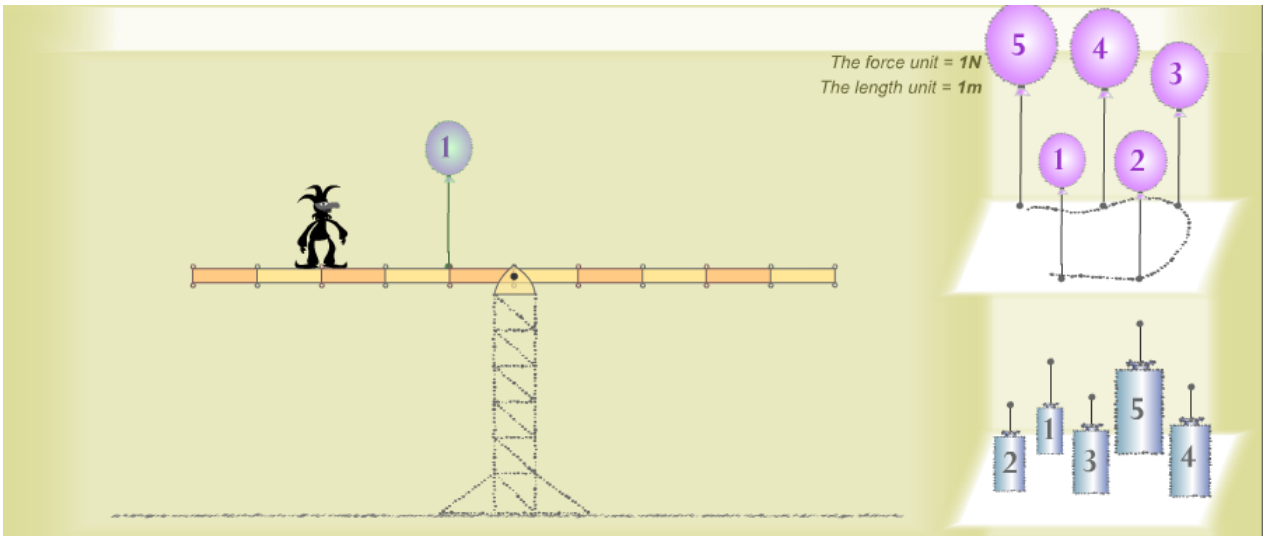
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The length unit = 1m
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4

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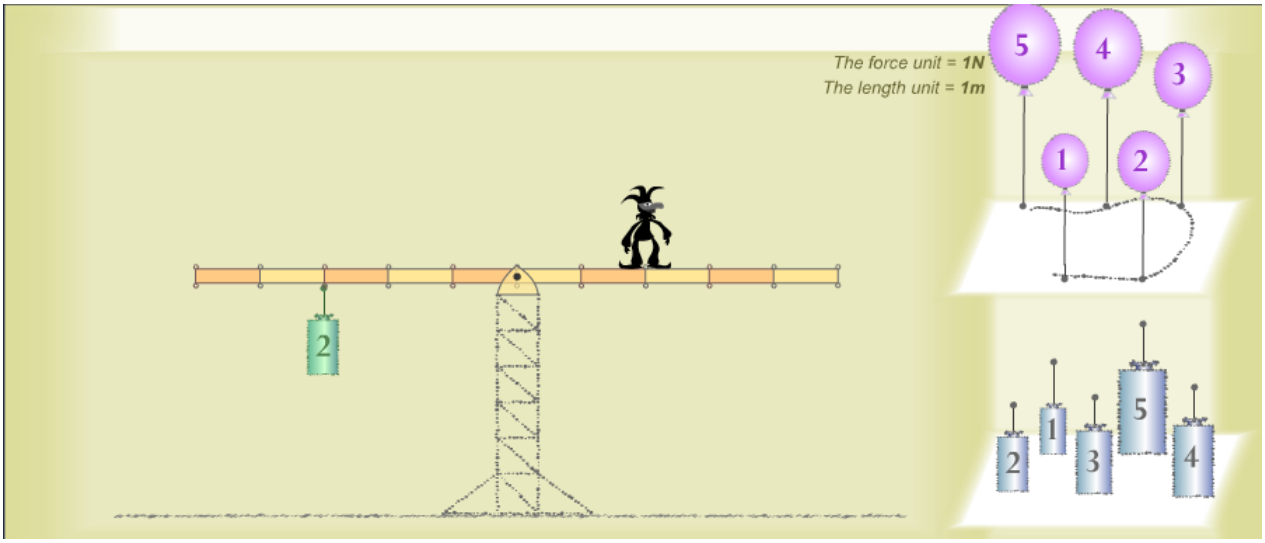
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The length unit = 1m
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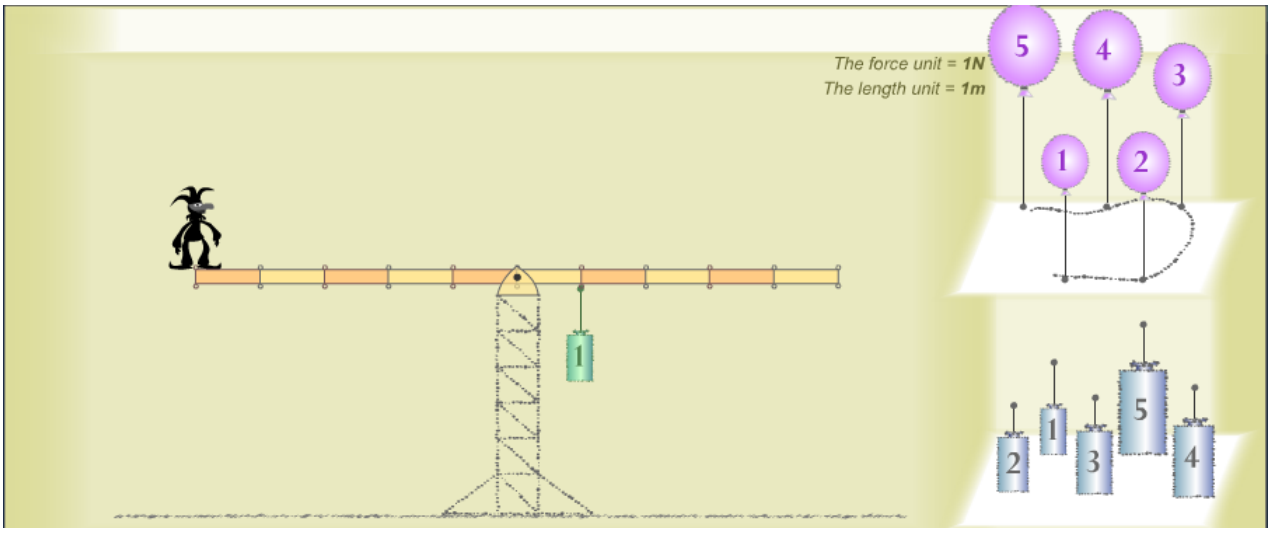
The force unit = 1N
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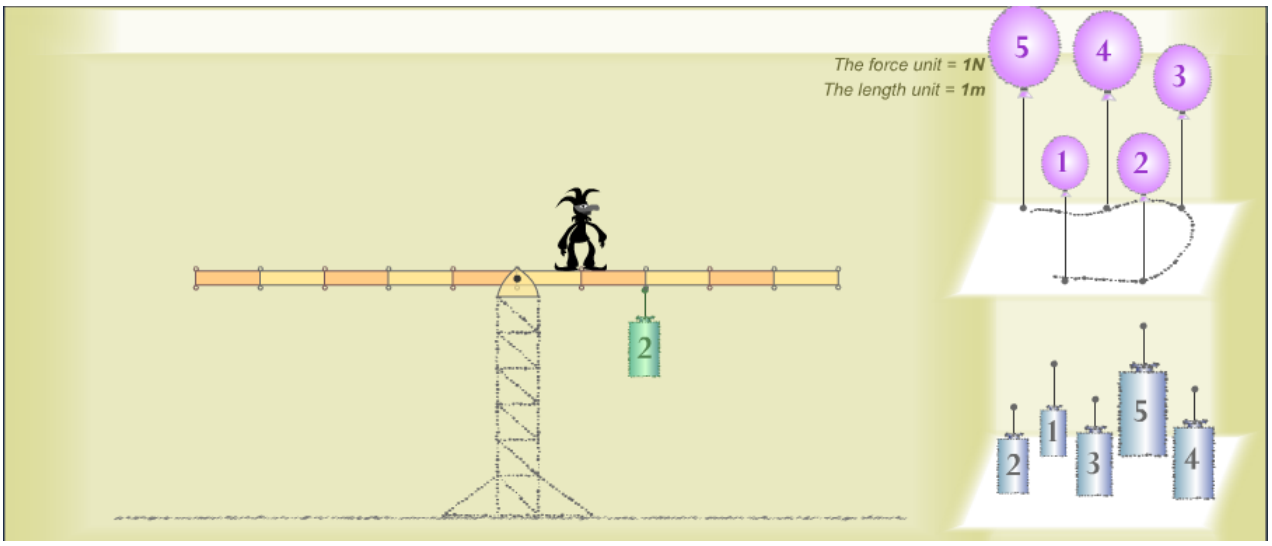
The force unit = 1N
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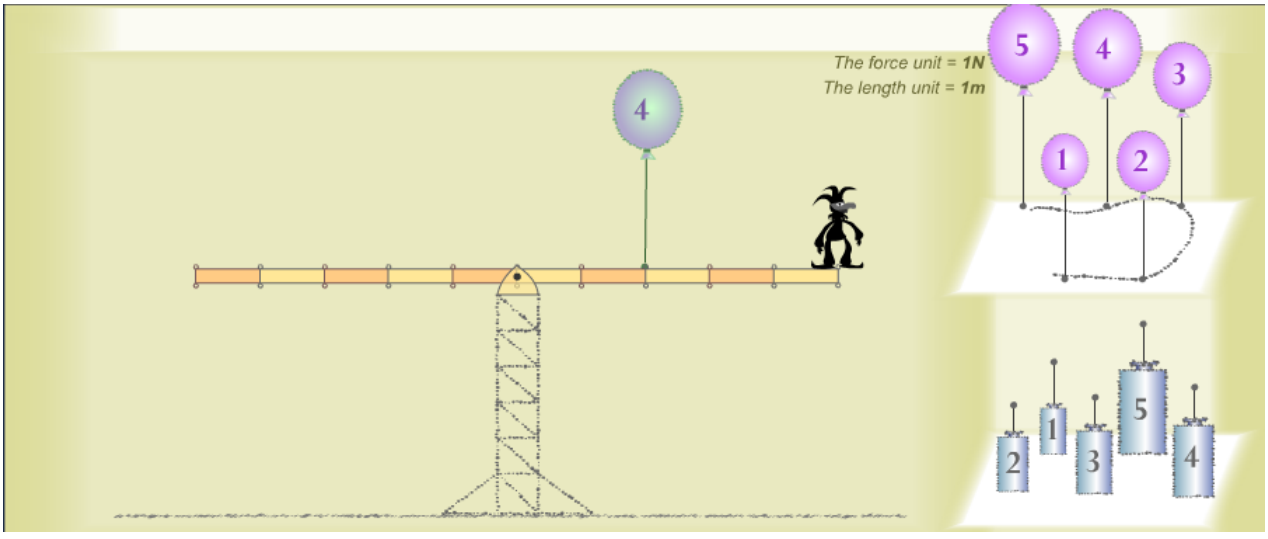
The force unit = 1N
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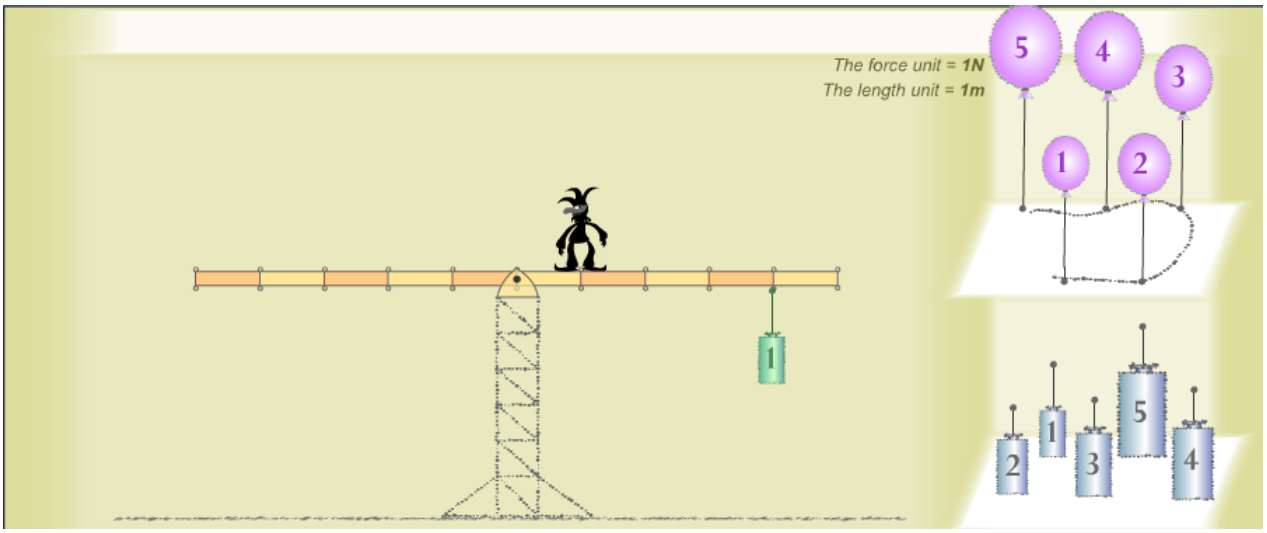
The force unit = 1N
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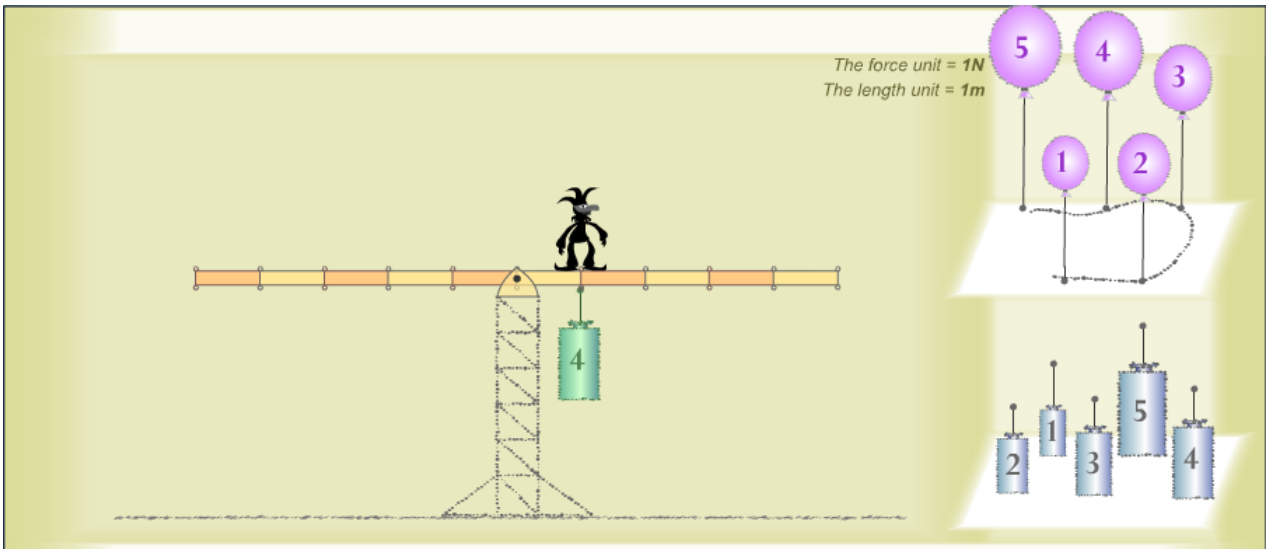
The force unit = 1N
The length unit = 1m
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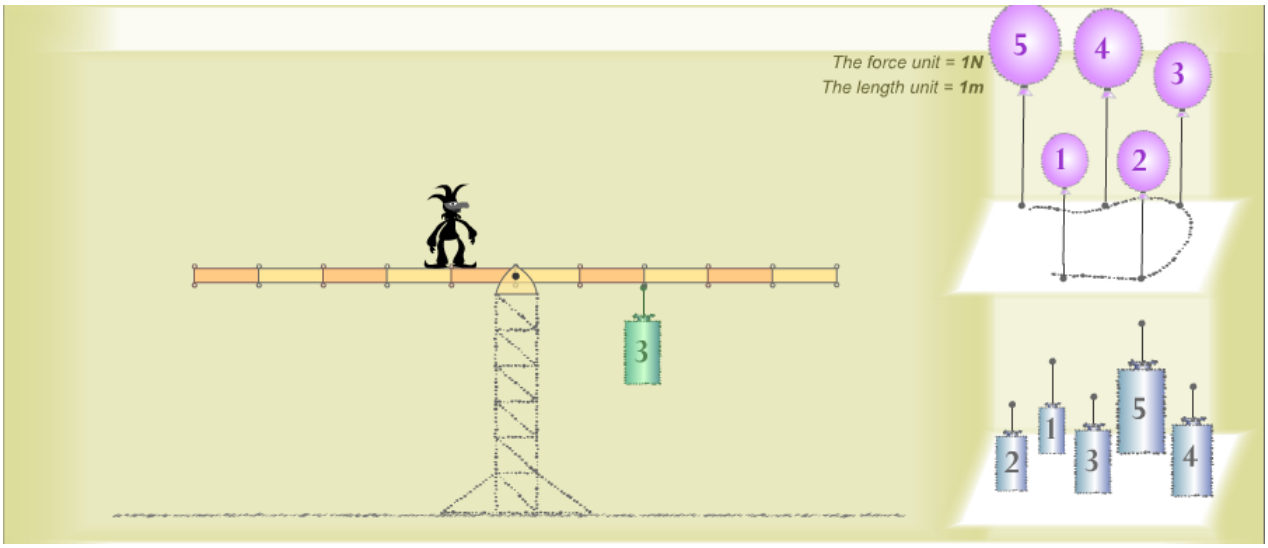
The force unit = 1N
The length unit = 1m
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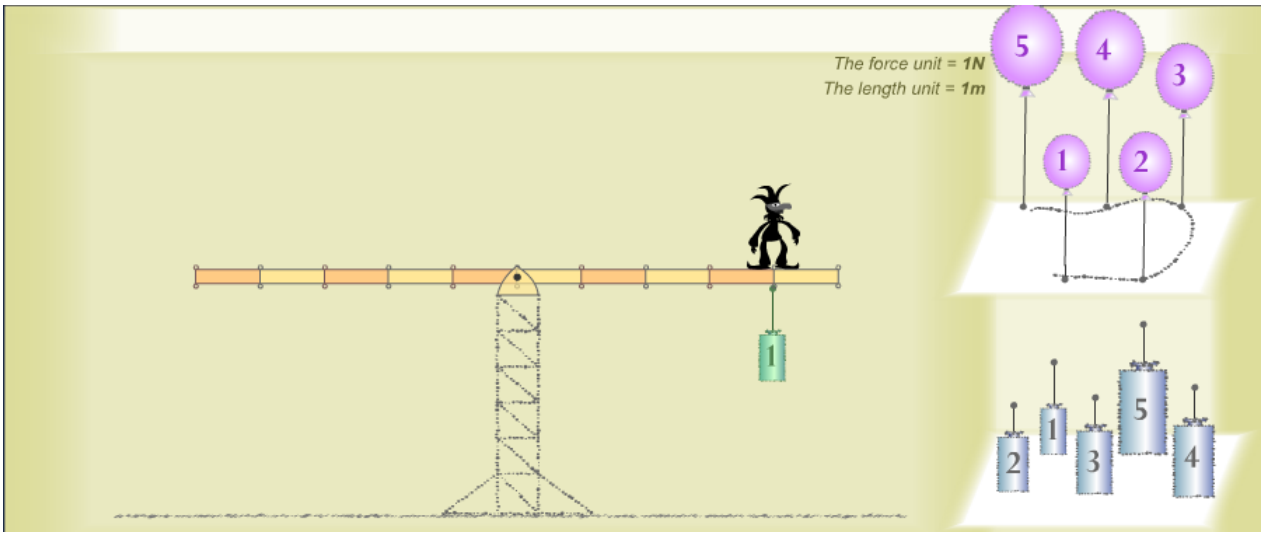
The force unit = 1N
The length unit = 1m
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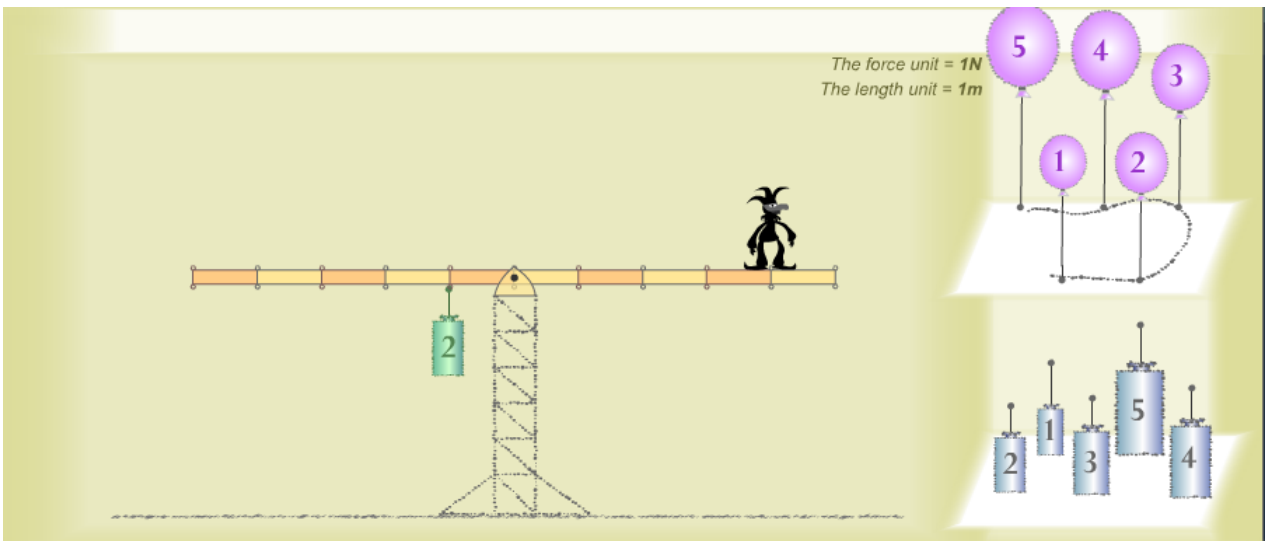
The force unit = 1N
The length unit = 1m
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