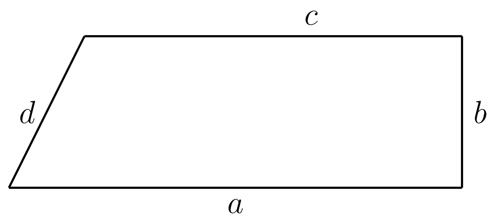
More Geometry

Nakul and Andreas

Theorem 1 (Triangle inequality)**.**
For any three points A, B, C, $AB + BC \geq AC$. Moreover, if B does not belong to the segment AC, the inequality is strict.

Problem 1.
A trapezoid has sides of lengths 2, 4, 5 and 6. Which of them may be parallel?



Problem 2.
Let P be a point given in the interior of a circle. The smallest distance from P to the circumference of the circle is a and the largest is b. Find the radius of the circle.

Problem 3.
Let P and Q be points given in the interior of a given angle. Find points A and B on the sides of the angle such that $PA + AQ + QB + BP$ is minimized. You can interpret this as a path from P back to itself, touching both sides and the other point, but in a specific order. (First a side, then point Q, then the other side and then back to P.)

Problem 4.
Three hundred people live in village A, five hundred in village B and one thousand in village C. Where should the state build a single hospital for these villages so that the total commute time would be minimal? (Hint: Formally, minimize $300|AP| + 500|BP| + 1000|CP|$.)

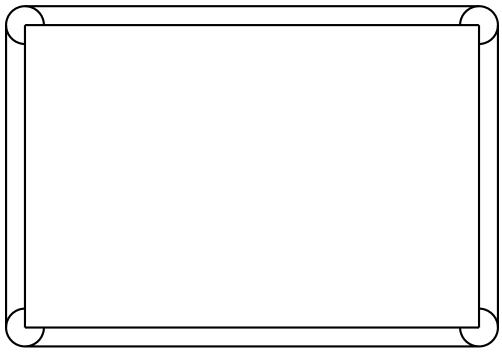
Problem 5.
Striking grad students want to form a rectangular picket line that encloses as much area as possible, given the total length of the picket line $2a + b$ can be as big as 100 meters. What is the optimal way to do so?

Problem 6.
Given points A, B and C, find point P such that $|AP|^2 + |BP|^2 + |CP|^2$ is minimized.

Problem 7.
Show that the set of all points, such that any three form a triangle of area at most 1, must be contained in a triangle of area 4.

Problem 8.
Find a way to connect the four vertices and the center of a 4×4 square with a road network such that the total road length is < 11.

Problem 9.
A rectangular pool table with one pocket on each corner has side lengths 4 feet and 6 feet. A ball is placed at one corner and aimed at an angle of $45°$. A player wants to determine how much strength he needs to hit the ball with so that it eventually lands in a pocket.



(a) How far does the ball need to travel before it reaches a pocket (i.e. a corner)?

(b) Repeat part a) for side lengths 3 feet and 5 feet. Can you generalize your answer for side lengths a and b?

(c) Now assume that a very skilled carpenter has created a table of lengths 1 foot and $\sqrt{3}$ feet. Find a formula for the angle(s) the ball can be shot at so that it eventually reaches a pocket.