3 Problems

What is the value of the product

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdot\left(1+\frac{1}{5}\right)\cdot\left(1+\frac{1}{6}\right)?$$

(A) $\dfrac{7}{6}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{7}{2}$ (D) 7 (E) 8

AMC 8 2018, Problem 2

Four numbers are written in a row. The average of the first two is $21,$ the average of the middle two is $26,$ and the average of the last two is 30. What is the average of the first and last of the numbers?

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

AMC 8 2022, Problem 16

Each of the points $A, B, C, D, E,$ and F in the figure below represents a different digit from 1 to 6. Each of the five lines shown passes through some of these points. The digits along each line are added to produce five sums, one for each line. The total of the five sums is 47. What is the digit represented by B?
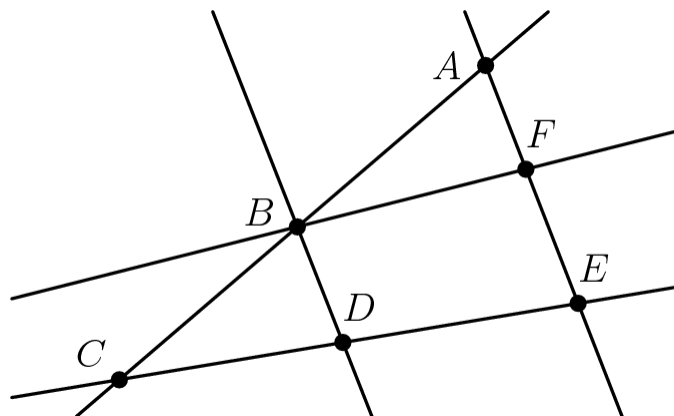
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

AMC 8 2020, Problem 16

What is the value of the product $\left(\dfrac{1 \cdot 3}{2 \cdot 2}\right) \left(\dfrac{2 \cdot 4}{3 \cdot 3}\right) \left(\dfrac{3 \cdot 5}{4 \cdot 4}\right) \cdots \left(\dfrac{97 \cdot 99}{98 \cdot 98}\right) \left(\dfrac{98 \cdot 100}{99 \cdot 99}\right)$?

(A) $\dfrac{1}{2}$ (B) $\dfrac{50}{99}$ (C) $\dfrac{9800}{9801}$ (D) $\dfrac{100}{99}$ (E) 50

AMC 8 2019, Problem 17

An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

(A) 21 (B) 31 (C) 36 (D) 40 (E) 42

1				25
		X		
17				81

AMC 8 2015, Problem 18

Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from the highest score to the lowest score?

(A) Hannah, Cassie, Bridget   (B) Hannah, Bridget, Cassie
(C) Cassie, Bridget, Hannah   (D) Cassie, Hannah, Bridget
(E) Bridget, Cassie, Hannah

AMC 8 2013, Problem 19

The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number x in the lower left corner is larger than the other three missing

-2	9	5
		-1
x		8

numbers. What is the smallest possible value of x?

(A) -1 (B) 5 (C) 6 (D) 8 (E) 9

AMC 8 2022, Problem 20

A game board consists of 64 squares that alternate in color between black and white. The figure below shows square P in the bottom row and square Q in the top row. A marker is placed at P. A step consists of moving the marker onto one of the adjoining white squares in the row above. How many 7-step paths are there from P to Q? (The figure shows a sample path.)
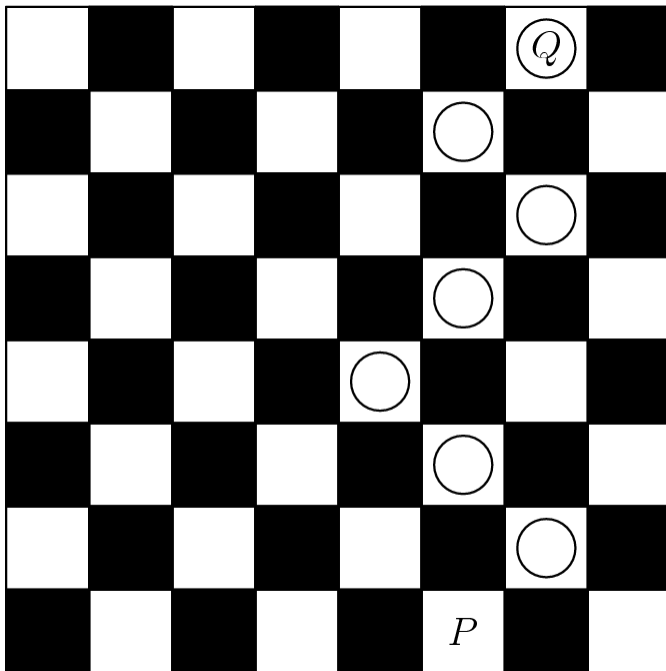
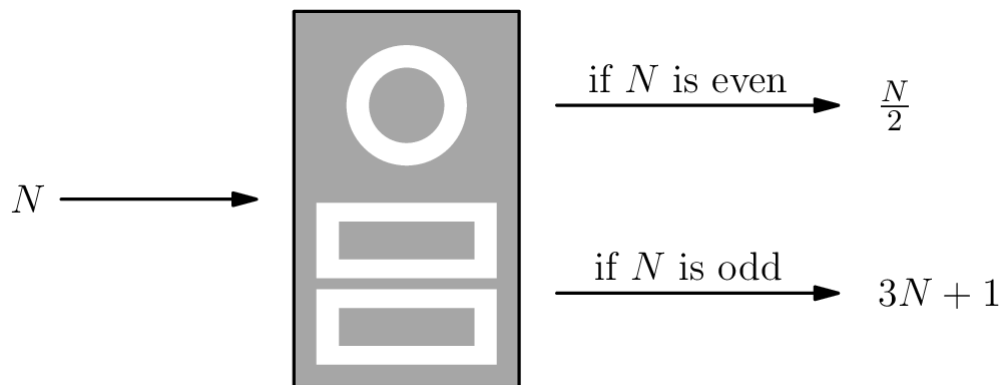(A) 28   (B) 30   (C) 32   (D) 33   (E) 35

Suppose a, b, and c are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for $\dfrac{a}{|a|} + \dfrac{b}{|b|} + \dfrac{c}{|c|} + \dfrac{abc}{|abc|}$?

(A) 0 (B) 1 and -1 (C) 2 and -2 (D) $0, 2,$ and -2 (E) $0, 1,$ and -1

AMC 8 2017, Problem 21

When a positive integer N is fed into a machine, the output is a number calculated according to the rule shown below.

if N is even $\longrightarrow \dfrac{N}{2}$

if N is odd $\longrightarrow 3N + 1$

For example, starting with an input of $N = 7$, the machine will output $3 \cdot 7 + 1 = 22$. Then if the output is repeatedly inserted into the machine five more times, the final output is $26.7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26$ When the same 6-step process is applied to a different starting value of N, the final output is 1. What is the sum of all such integers N? $N \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to 1$

(A) 73 (B) 74 (C) 75 (D) 82 (E) 83

AMC 8 2020, Problem 22

Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

(A) 10 (B) 15 (C) 25 (D) 50 (E) 82

AMC 8 2017, Problem 23

Three members of the Euclid Middle School girls' softball team had the following conversation.

Ashley: I just realized that our uniform numbers are all 2-digit primes.

Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.

Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Ashley: And the sum of your two uniform numbers is today's date.

What number does Caitlin wear?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 13 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 23$

AMC 8 2014, Problem 23

The least positive integer with exactly 2021 distinct positive divisors can be written in the form $m \cdot 6^k$, where m and k are integers and 6 is not a divisor of m. What is $m + k$?

$\textbf{(A)}\ 47 \qquad \textbf{(B)}\ 58 \qquad \textbf{(C)}\ 59 \qquad \textbf{(D)}\ 88 \qquad \textbf{(E)}\ 90$

Hint: $2021 = 43 \times 47$

AMC 10B 2021 Fall, Problem 6

A two-digit positive integer is said to be $cuddly$ if it is equal to the sum of its nonzero tens digit and the square of its units digit. How many two-digit positive integers are cuddly?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

AMC 10A 2021 Fall, Problem 8