ORMC Olympiad Group
Fall: Week 6
Geometry: Circles

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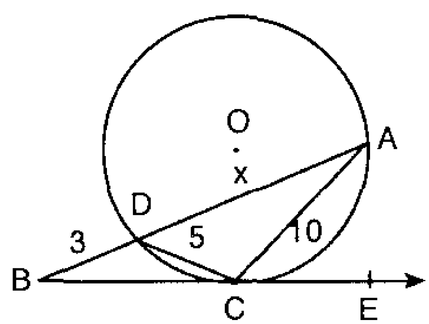
Problems

1. The circle with center O has diameter length $\sqrt{2425}$. Two chords AB and CD have midpoints M and N respectively. If $CD - AB = OM - ON = 2$, find the area of the triangle ODC.

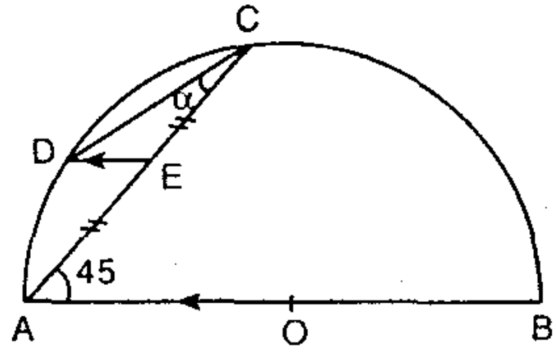
2. The angle bisector of $\angle BAC$ of triangle ABC cuts the circumcircle at point M. Let I be the incenter. Prove that $MB = MI = MC$.

3. **(ZG)** In the figure below the circle is tangent to the line BE at point C. If $BD = 3, DC = 5, CA = 10$, find $AD = x$.



4. **(ZG)** In the following figure below point C is chose on the half circle with center O such that $\angle OAC = 45$. AB is the diameter and E is

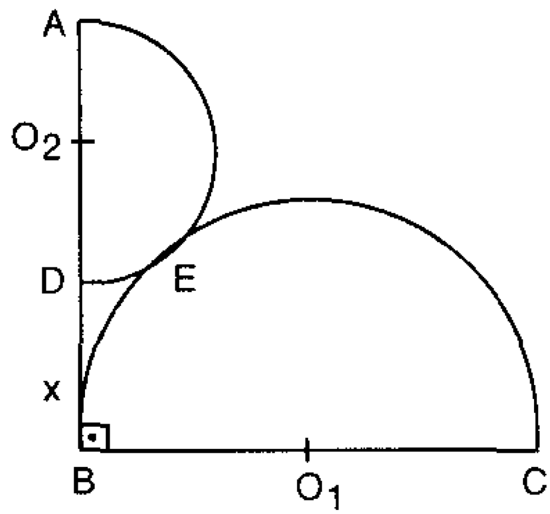
midpoint of AC. D is on the minor arc $\overset{\frown}{AC}$ such that $ED \parallel AB$. Find $\angle DCA = \alpha$.



5. **(ZG-modified)** In the following figure below O_1 and O_2 are the centers, two circles are tangent at point E and $AB = BC = 12$.

   (i) Compute $DB = x$.

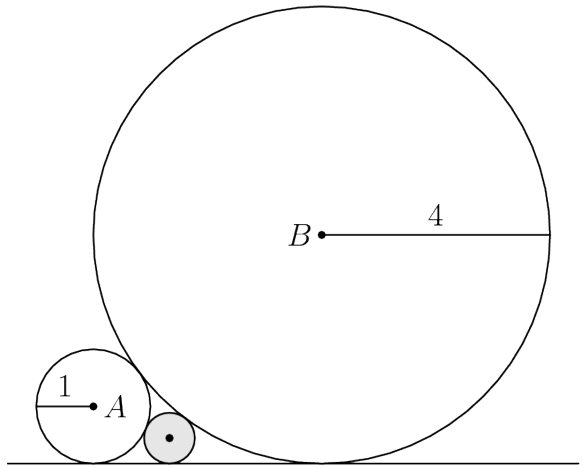
   (ii) Compute ED.



6. **(LAMC)** Two circles of radii R and r respectively are tangent to each other. A line l is tangent to both circles, at points A and B respectively. Find the length of the segment AB in terms of R and r.

7. **(AMC12 2001)** A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle

is tangent to the first two and to one of their common external tangents as shown. That is the radius of the third circle?



8. Let O be the center of the circle Γ with radius 150. Let P be a point outside of the circle such that $OP = 250$. The tangents PT and PS are drawn so that T and S are on the circle. Point Q is chosen on the segment PT such that $SQ \perp TP$. SQ intersects with the circle S again at point X. What ia the length of the segment QX.

9. Let ABC be acute triangle and AH is an altitude with $H \in [BC]$. $AH = 4, BH = 3, HC = \frac{1}{3}$, and the circumcircle of AHC cuts the side AB at D. Chose K on the side BC so that $BK = 1$, and DK intersects the circumcircle of AHC again at L. The length of KL can be written as $\frac{a\sqrt{b}}{c}$ where a, b, c are positive integers, b is square-free and $(a, c) = 1$. What is $a + b + c$?

10. **(9-point Circle)** Let ABC be a triangle with orthocenter H. Let H_a, H_b and H_c be feet of the altitudes from vertices A, B, C to the opposide sides. Let M_a, M_b and M_c be midpoints of the sides BC, AC and AB. Let X, Y and Z be midpoints of segments AH, BH and CH. Prove that said 9 points $H_a, H_b, H_c, M_a, M_b, M_c, X, Y, Z$ lie on a circle. This circle is called $9 - \textit{point circle}$.

    Additional Question: What can you say about center of this circle? How about radious?

11. **(Simpson's Line)** Let ABC be a triangle. P is a point on the plane

of ABC. Let X, Y, Z be the feet of the perpendiculars from point P to the lines BC, AC and AB respectively. Then following are equivalent

(a) P lies on the circumcircle of ABC

(b) X, Y, Z are collinear.

12. **(LAMC 2008 - modified)** Let AB and CD be two diameters of a circle with center O and radius 12. Assume $\angle AOC = 45°$. Let M be a point on the circle, and points P and Q are chosen on the diameters AB and CD so that $\angle MQA = \angle MPB = 60°$. Compute PQ.

13. (AIME-II 2013 8 - modified) A hexagon that is inscribed in a circle has side lengths 10, 10, 20, 10, 10, and 20 in that order. The radius of the circle can be written as $p + q\sqrt{r}$, where p, q and r are positive integers, r is square-free integer. Find $p + q + r$.

14. **(Turkey JBMO TST-2013)** Let D be a point on the side BC of an equilateral triangle ABC where D is different than the vertices. Let I be the excenter of the triangle ABD opposite to the side AB and J be the excenter of the triangle ACD opposite to the side AC. Let E be the second intersection point of the circumcircles of triangles AIB and AJC. Prove that A is the incenter of the triangle IEJ.

15. **(IMO 2018)** Let Γ be the circumcircle of acute triangle ABC. Points D and E are on segments AB and AC respectively such that $AD = AE$. The perpendicular bisectors of BD and CE intersect minor arcs AB and AC of Γ at points F and G respectively. Prove that lines DE and FG are either parallel or they are the same line.