Binary Numbers Overview
Spring 2012, Meeting 4

4/29/12

Addition

For the following addition problems, do the following:

- Convert the numbers into binary notation;
- Add the numbers in binary notation;
- Convert the answer into decimal notation;
- Check the addition in decimal notation.

1. $5 + 7 =$

2. $10 + 10 =$

3. $15 + 13 =$

Subtraction

For the following addition problems, do the following:

- Convert the numbers into binary notation;
- Subtract the numbers in binary notation;
- Convert the answer into decimal notation;
- Check the subtraction in decimal notation.

1. $7 - 5 =$

2. $6 - 5 =$

3. $13 - 7 =$

4. What is the *largest* number that can be written in binary notation using two 1s and a 0?

   Convert this number into decimal notation:

5. What is the *smallest* number that can be written in binary notation using two 1s and a 0?

   Convert this number into decimal notation:

6. What is the *largest* number that can be written in binary notation using two 1s and two 0s?

   Convert this number into decimal notation:

7. What is the *smallest* number that can be written in binary notation using two 1s and two 0s?

   Convert this number into decimal notation:

8. For the two numbers written in binary notation below, decide which one is even and which one is odd:

$$1001***10$$

$$1001***01$$

Here $***$ stand for any binary digits.

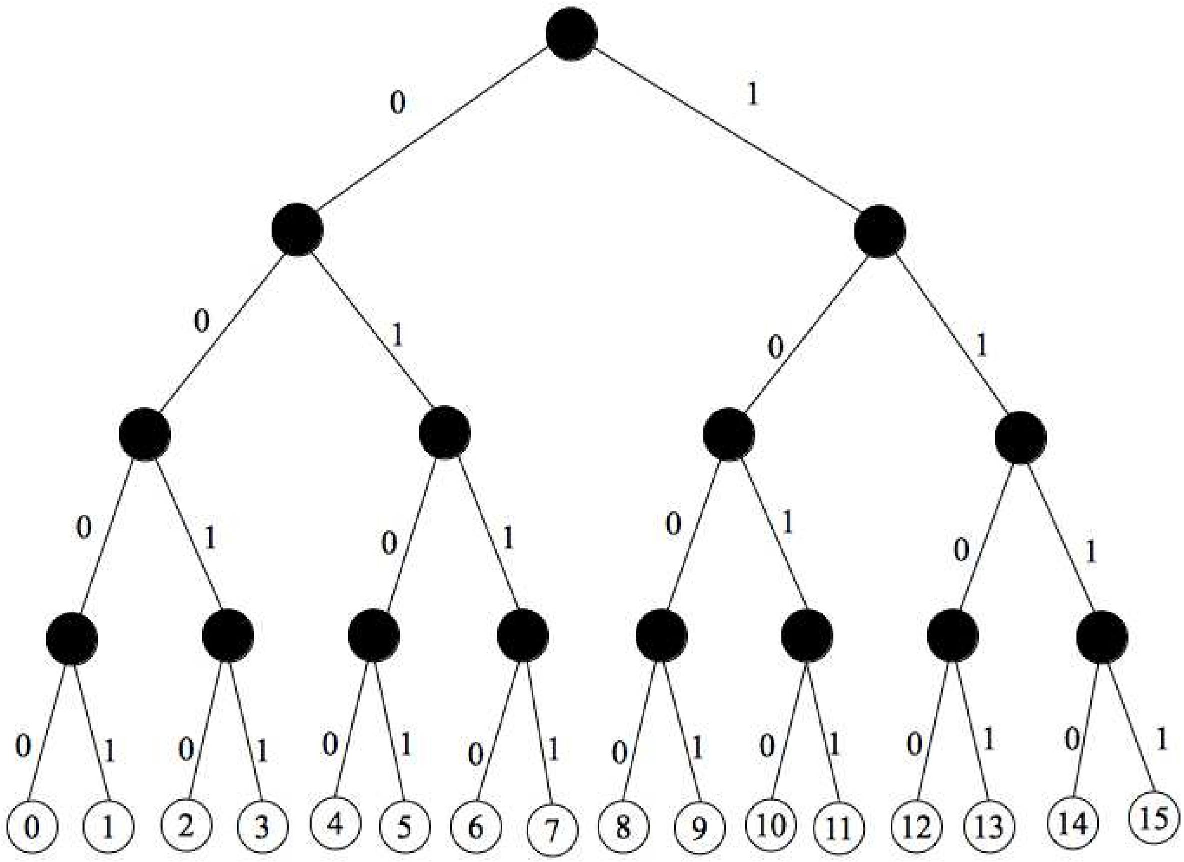
9. Given the powers of 2:

$$2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128, ...$$

convert the following binary numbers into decimal notation:

- $(100)_2 =$, $(11)_2 =$, $(101)_2 =$;
- $(1000)_2 =$, $(111)_2 =$, $(1001)_2 =$;
- $(10000)_2 =$, $(1111)_2 =$, $(10001)_2 =$;
- What should the next question be?

10. Use the binary tree below to figure out the binary notation for the numbers below:



- 13 =
- 7 =
- 15 =
- 9 =

More on the card trick

Recall binary representation of numbers $1 - 15$:

Number	The number as the sum of powers of 2	8	4	2	1
1	$1 =$	0	0	0	1
2	$2 =$	0	0	1	0
3	$3 = 2 + 1$				
4	$4 =$	0	1	0	0
5	$5 =$				
6	$6 = 4 + 2$	0	1	1	0
7	$7 =$				
8	$8 =$				
9	9	1	0	0	1
10	$10 =$				
11	$11 =$				
12	$12 =$				
13	$13 =$	1	1	0	1
14	$14 =$				
15	$15 =$				

Now recall the cards used in the magic trick game:

$$\mathbf{I} = \begin{pmatrix} 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{pmatrix} \quad \mathbf{II} = \begin{pmatrix} 4 & 5 & 6 & 7 \\ 12 & 13 & 14 & 15 \end{pmatrix}$$
$$\mathbf{III} = \begin{pmatrix} 2 & 3 & 6 & 7 \\ 10 & 11 & 14 & 15 \end{pmatrix} \quad \mathbf{IV} = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 9 & 11 & 13 & 15 \end{pmatrix}$$

1. Look at all the numbers on card I. What is their first digit?
2. What is the common digit of the numbers on card II?
3. What is the common digit of the numbers on card III?
4. What is the common digit of the numbers on card IV?

5. Given a number, how do you know on which cards it appears? (*Hint:* look at the binary digits of this number):

   - Without looking at the cards, decide on which cards will the number 6 appear? (First, look up the binary notation for 6 in the table above). Then, look at the cards to see if you were right.

   - Without looking at the cards, decide on which cards will the number 9 appear? (First, look up the binary notation for 9 in the table above). Then, look at the cards to see if you were right.

   - Without looking at the cards, decide on which cards will the number 12 appear? (First, look up the binary notation for 12 in the table above). Then, look at the cards to see if you were right.

   - Without looking at the cards, decide on which cards will the number 3 appear? (First, look up the binary notation for 3 in the table above). Then, look at the cards to see if you were right.

6. **Homework!**

   (a) Make cards for the card trick game that we played earlier today with the numbers 1-7.
   How many cards will you need?

   (b) Learn to play the card trick game (both for the numbers $1-7$ and $1-15$) and play it with your family and friends.