

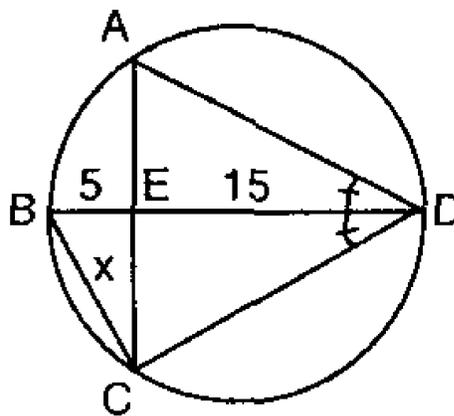
ORMC Olympiad Group
 Fall: Week 6
 Geometry: Circles

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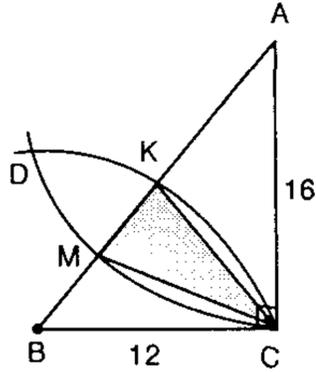
Problems

- (ZG) In the figure below the diameter BD is also angle bisector of $\angle ADC$. It is given $BE = 5, ED = 15$. Find $BC = x$.

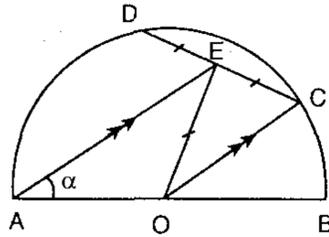


- (ZG-modified) In the figure the right triangle ABC has sides $BC = 12, CA = 16$. The circle with center A and radius AC cuts AB again at M , and similarly the circle with center B and radius BC cuts AB again at K .
 - Find the area of the triangle KMC .

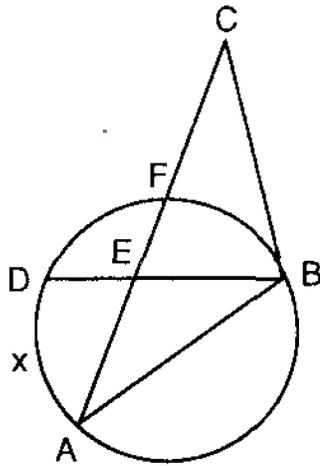
(ii) Find $\sin \angle KCM$.



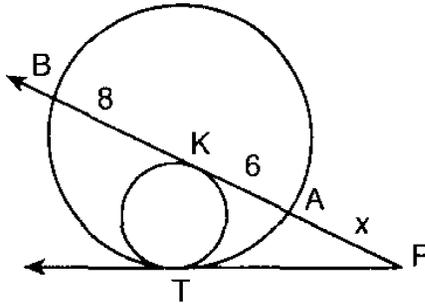
3. (ZG) In the following figure below O is the center, E is the midpoint of CD , and $AE \parallel OC$. If $OE = EC$, find $\angle BAE = \alpha$.



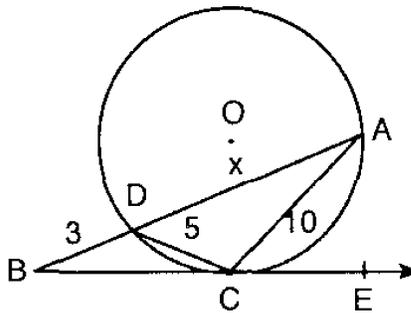
4. (ZG) In the following figure below $\widehat{DF} = 90$, $\widehat{FB} = 60$, and CB is tangent to the circle. If $\angle ACB = 50$, find $\widehat{AD} = x$.



5. (LAMC 2008) Let $ABCD$ be a quadrilateral inscribed in a circle with center O . Let M be the middle of the arc AB . Let $E = MC \cap AB$ and $K = MD \cap AB$. Show that the quadrilateral $KECD$ is cyclic.
6. (ZG) In the following figure below two circles are tangent to each other and line PT at point T . If $BK = 8, KA = 6$, find $AP = x$.

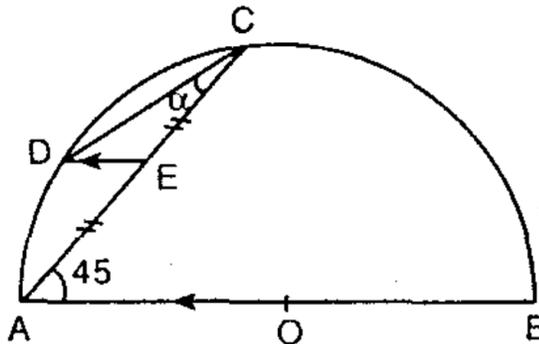


7. $ABCD$ is cyclic quadrilateral with $AB = 1, BC = 2, CD = 3, DA = 4$. What is AC^2 ?
8. The circle with center O has diameter length $\sqrt{2425}$. Two chords AB and CD have midpoints M and N respectively. If $CD - AB = OM - ON = 2$, find the area of the triangle ODC .
9. The angle bisector of $\angle BAC$ of triangle ABC cuts the circumcircle at point M . Let I be the incenter. Prove that $MB = MI = MC$.
10. (ZG) In the figure below the circle is tangent to the line BE at point C . If $BD = 3, DC = 5, CA = 10$, find $AD = x$.



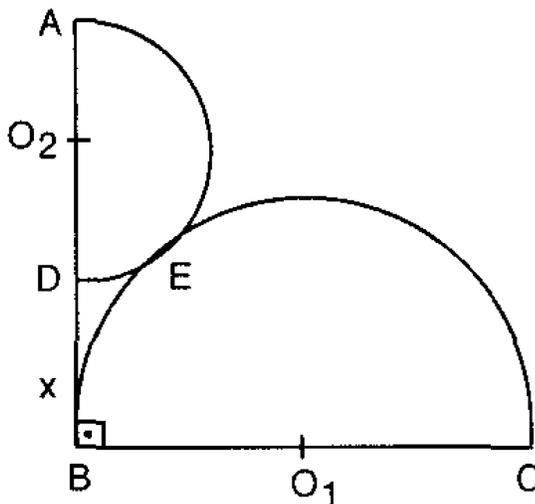
11. (ZG) In the following figure below point C is chose on the half circle with center O such that $\angle OAC = 45$. AB is the diameter and E is

midpoint of AC . D is on the minor arc \widehat{AC} such that $ED \parallel AB$. Find $\angle DCA = \alpha$.



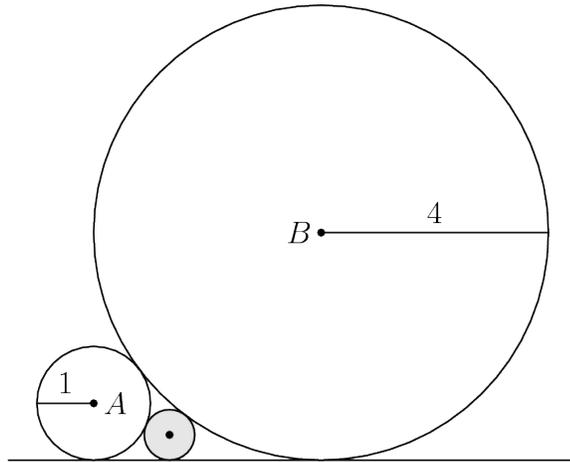
12. **(ZG-modified)** In the following figure below O_1 and O_2 are the centers, two circles are tangent at point E and $AB = BC = 12$.

- (i) Compute $DB = x$.
- (ii) Compute ED .



- 13. **(LAMC)** Two circles of radii R and r respectively are tangent to each other. A line l is tangent to both circles, at points A and B respectively. Find the length of the segment AB in terms of R and r .
- 14. **(AMC12 2001)** A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle

is tangent to the first two and to one of their common external tangents as shown. That is the radius of the third circle?



15. Let O be the center of the circle Γ with radius 150. Let P be a point outside of the circle such that $OP = 250$. The tangents PT and PS are drawn so that T and S are on the circle. Point Q is chosen on the segment PT such that $SQ \perp TP$. SQ intersects with the circle S again at point X . What is the length of the segment QX .
16. Let ABC be acute triangle and AH is an altitude with $H \in [BC]$. $AH = 4$, $BH = 3$, $HC = \frac{1}{3}$, and the circumcircle of AHC cuts the side AB at D . Chose K on the side BC so that $BK = 1$, and DK intersects the circumcircle of AHC again at L . The length of KL can be written as $\frac{a\sqrt{b}}{c}$ where a, b, c are positive integers, b is square-free and $(a, c) = 1$. What is $a + b + c$?