Problems

1. *(ZG)* In the figure below the diameter $BD$ is also angle bisector of $\angle ADC$. It is given $BE = 5$, $ED = 15$. Find $BC = x$.

   ![Diagram](image)

2. *(ZG-modified)* In the figure the right triangle $ABC$ has sides $BC = 12$, $CA = 16$. The circle with center $A$ and radius $AC$ cuts $AB$ again at $M$, and similarly the circle with center $B$ and radius $BC$ cuts $AB$ again at $K$.

   (i) Find the area of the triangle $KMC$. 
(ii) Find \( \sin \angle KCM \).

3. (ZG) In the following figure below \( O \) is the center, \( E \) is the midpoint of \( CD \), and \( AE \parallel OC \). If \( OE = EC \), find \( \angle BAE = \alpha \).

4. (ZG) In the following figure below \( \widehat{DF} = 90, \widehat{FB} = 60 \), and \( CB \) is tangent to the circle. If \( \angle ACB = 50 \), find \( \widehat{AD} = x \).
5. **(LAMC 2008)** Let $ABCD$ be a quadrilateral inscribed in a circle with center $O$. Let $M$ be the middle of the arc $AB$. Let $E = MC \cap AB$ and $K = MD \cap AB$. Show that the quadrilateral $KECD$ is cyclic.

6. **(ZG)** In the following figure below two circles are tangent to each other and line $PT$ at point $T$. If $BK = 8$, $KA = 6$, find $AP = x$.

![Diagram](image)

7. $ABCD$ is cyclic quadrilateral with $AB = 1$, $BC = 2$, $CD = 3$, $DA = 4$. What is $AC^2$?

8. The circle with center $O$ has diameter length $\sqrt{2425}$. Two chords $AB$ and $CD$ have midpoints $M$ and $N$ respectively. If $CD - AB = OM - ON = 2$, find the area of the triangle $ODC$.

9. The angle bisector of $\angle BAC$ of triangle $ABC$ cuts the circumcircle at point $M$. Let $I$ be the incenter. Prove that $MB = MI = MC$.

10. **(ZG)** In the figure below the circle is tangent to the line $BE$ at point $C$. If $BD = 3$, $DC = 5$, $CA = 10$, find $AD = x$.

![Diagram](image)

11. **(ZG)** In the following figure below point $C$ is chose on the half circle with center $O$ such that $\angle OAC = 45$. $AB$ is the diameter and $E$ is
midpoint of $AC$. $D$ is on the minor arc $\widehat{AC}$ such that $ED \parallel AB$. Find $\angle DCA = \alpha$.

12. (ZG-modified) In the following figure below $O_1$ and $O_2$ are the centers, two circles are tangent at point $E$ and $AB = BC = 12$.

(i) Compute $DB = x$.

(ii) Compute $ED$.

13. (LAMC) Two circles of radii $R$ and $r$ respectively are tangent to each other. A line $l$ is tangent to both circles, at points $A$ and $B$ respectively. Find the length of the segment $AB$ in terms of $R$ and $r$.

14. (AMC12 2001) A circle centered at $A$ with a radius of 1 and a circle centered at $B$ with a radius of 4 are externally tangent. A third circle
is tangent to the first two and to one of their common external tangents as shown. That is the radius of the third circle?

![Diagram showing circles and tangents](image)

15. Let $O$ be the center of the circle $\Gamma$ with radius 150. Let $P$ be a point outside of the circle such that $OP = 250$. The tangents $PT$ and $PS$ are drawn so that $T$ and $S$ are on the circle. Point $Q$ is chosen on the segment $PT$ such that $SQ \perp TP$. $SQ$ intersects with the circle $S$ again at point $X$. What is the length of the segment $QX$.

16. Let $ABC$ be acute triangle and $AH$ is an altitude with $H \in [BC]$. $AH = 4, BH = 3, HC = \frac{1}{3}$, and the circumcircle of $AHC$ cuts the side $AB$ at $D$. Chose $K$ on the side $BC$ so that $BK = 1$, and $DK$ intersects the circumcircle of $AHC$ again at $L$. The length of $KL$ can be written as $\frac{a\sqrt{b}}{c}$ where $a, b, c$ are positive integers, $b$ is square-free and $(a, c) = 1$. What is $a + b + c$?