LAMC Junior Circle **May 13, 2012**

Olga Radko
radko@math.ucla.edu

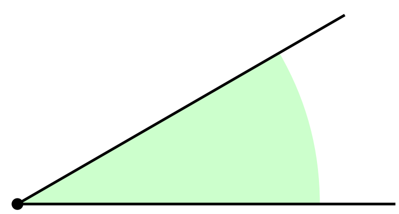
Oleg Gleizer
oleg1140@gmail.com

Problem 1 *Using a compass and a ruler, construct an equilateral triangle in the space below.*

Angles

A *ray* is a half of a straight line, finite in one direction, but infinite in the other. A ray contains its boundary point.

A (plane) *angle* is a plane figure formed by two rays with the common vertex and by all the points in between.



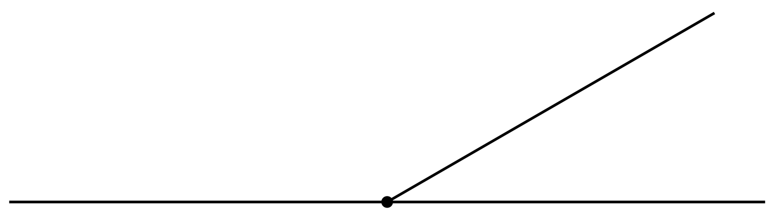
Looking at the above picture, we have to imagine that each of the two rays goes to infinity and so does the green coloring. This way, the rays split the plane into two parts, the green and white. The green-colored part, together with the boundary rays, is an angle. The white-colored region, bounded by the same rays, is also an angle. You have to specify the one you consider.

If the rays forming an angle lie on one and the same straight line, the angle is called a *straight angle*.

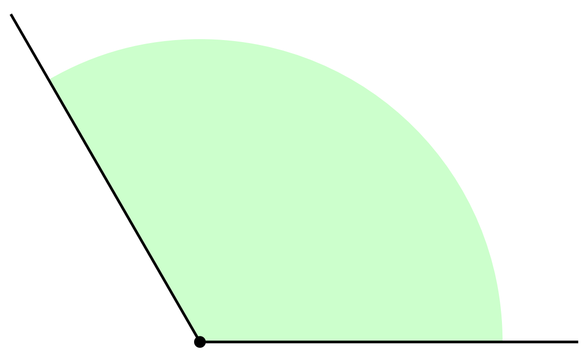
You have to imagine that the entire upper half-plane at the above picture is colored green. Note that the picture consists of two straight angles, the upper half-plane colored green and the lower half-plane colored white.

Problem 2 *Draw a straight line in the space below. How can you make it into a straight angle?*

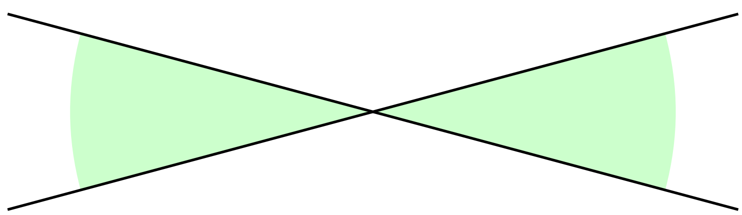
Two angles are called *supplementary*, if they add up to a straight angle.



Problem 3 *Using a ruler, draw an angle supplementary to the green-colored angle below. How many ways are there to solve this problem?*



Two angles are called *opposite*, if their sides form two straight lines as on the picture below.

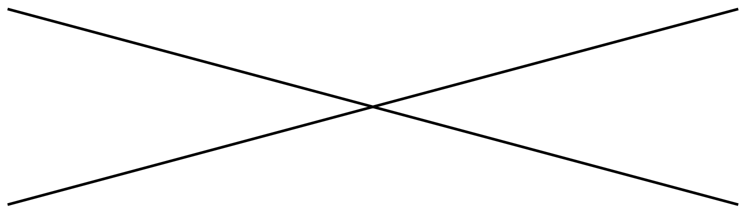


The green-colored angles at the above picture are opposite. The white-colored angles, supplementary to the green-colored angles, are opposite, too.

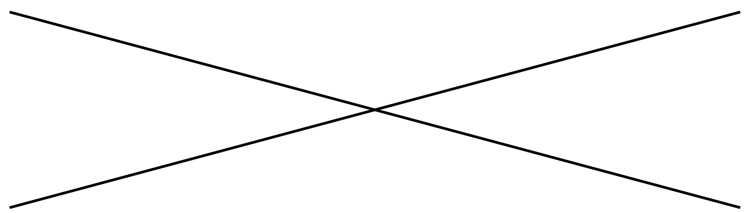
Proposition 1 *Opposite angles are equal to each other.*

In class, we will give two different proofs.

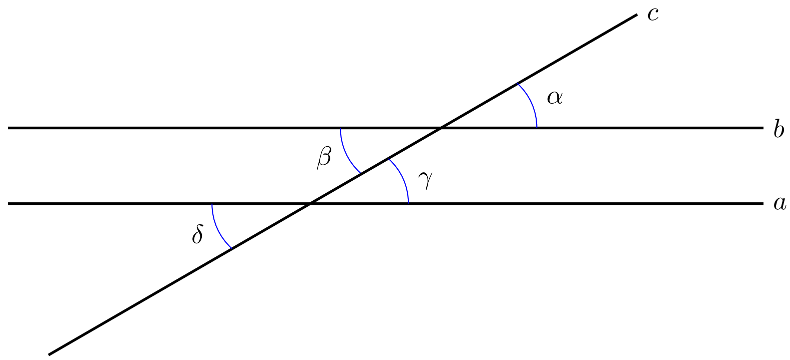
Problem 4 *Prove Proposition 1.*



Problem 5 *Give a different proof to Proposition 1.*

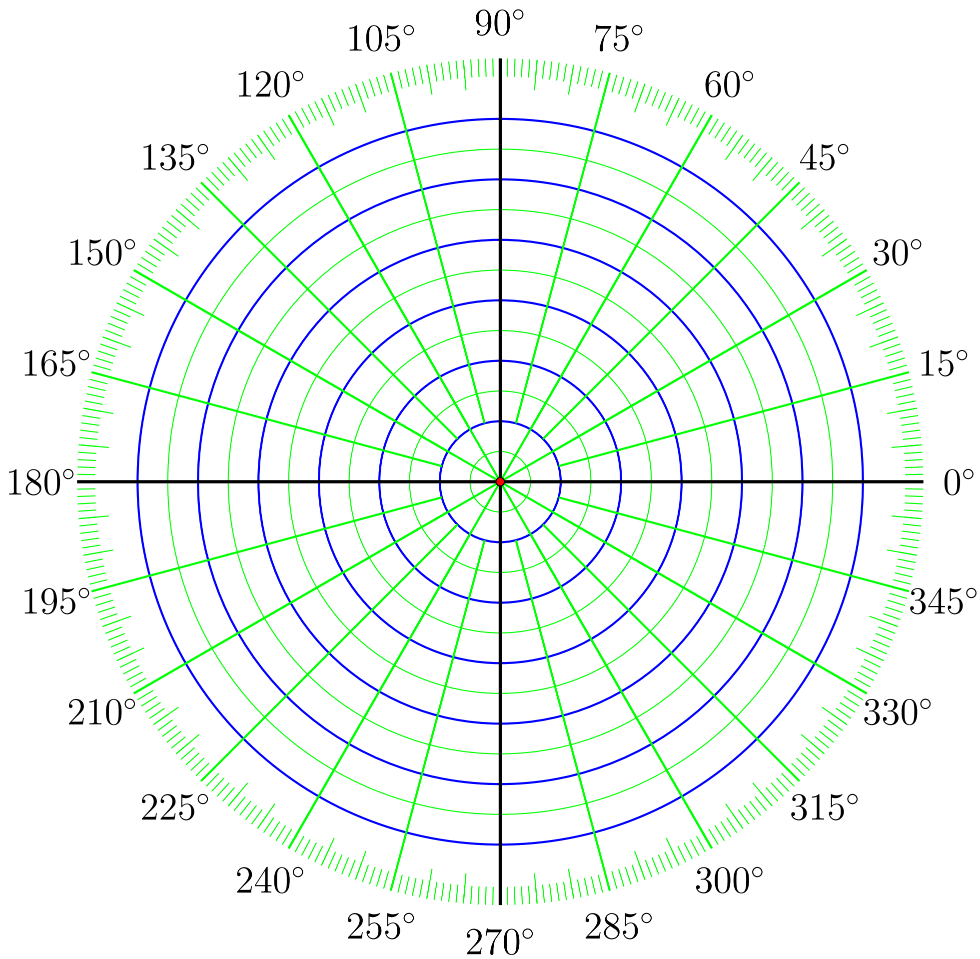


Proposition 2 *Let straight lines a and b be parallel. Let the straight line c intersect them as on the picture below. Then the angles α, β, γ, and δ are all equal to one another.*



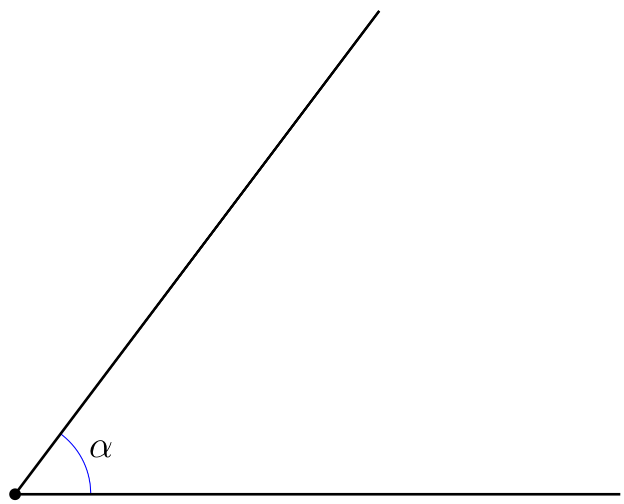
Problem 6 *Prove Proposition 2.*

A standard way to turn a circumference into a number line is to divide it into 360 equal parts. In this case, the unit step is called one (angular) *degree* (1°), the line with the marks, or a part of it, is called a *protractor*.



The TikZ code generating the above picture was written by Zoran Nikolic, downloaded from `http://www.texample.net/tikz/examples/polar-coordinates-template`, and slightly modified by one of the authors.

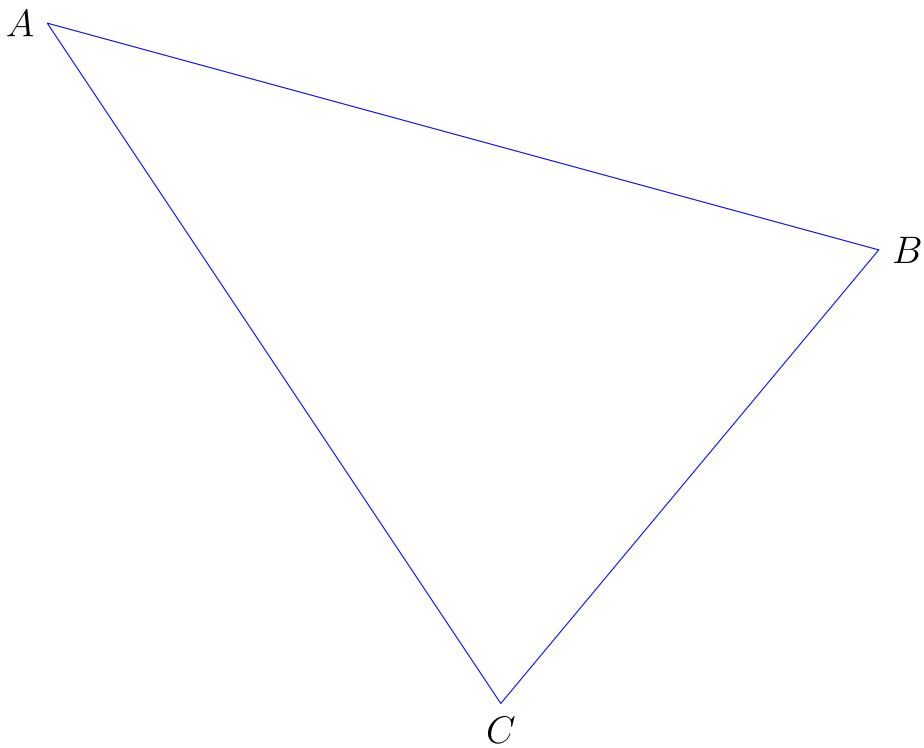
Problem 7 *Use a protractor to measure the following angle.*



$$\alpha =$$

Problem 8 *Use a ruler and a protractor to draw a* 75° *angle in the space below.*

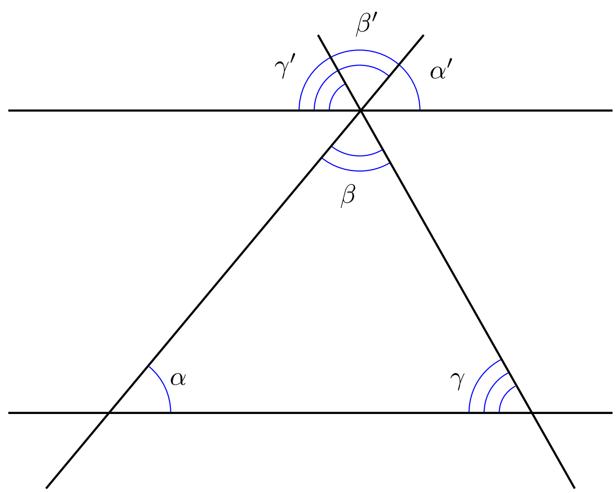
Problem 9 *Using a protractor, measure the angles $\angle A$, $\angle B$ and $\angle C$ of the below triangle. Find the sum $\angle A + \angle B + \angle C$.*



$$\angle A \ + \ \angle B \ + \ \angle C =$$

Theorem 1 *For any triangle in the Euclidean plane, its angles always add up to a straight angle.*

Problem 10 *Use the below picture to prove the theorem.*



Problem 11 *Without using a protractor, construct a* 60° *angle.*

Homework Problem 1 *Do the angles of any quadrilateral in the Euclidean plane always add up to some special angle? If you think they do, what is the size of the angle in degrees? Why?*