

ORMC Olympiad Group
Fall: Week 5
Geometry: Triangles and Circles

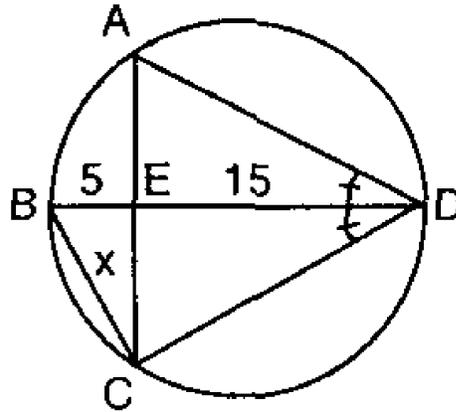
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Problems

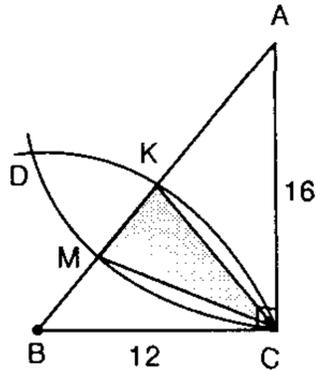
1. (**Prasolov 1.13**) In $\triangle ABC$ bisectors AA_1 and BB_1 are drawn. Prove that the distance from any point M of A_1B_1 to line AB is equal to the sum of distances from M to AC and BC .
2. $ABCD$ is a parallelogram, M is the midpoint of the side AD . Let H be the feet of the altitude from C to BM . Prove that $DH = CD$.
3. (**LAMC 2008**) Prove that in any triangle a median drawn to a side is smaller than half of the sum of the other two sides.
4. From a point A outside of the circle Γ the tangent AB is drawn, where B is the tangency point. Another line which passes through A cuts the circle Γ at points C and D . If $BC = 5$, $BD = 7$, what the maximum integer length that the segment AB can take?
5. (**ZG**) In the figure the diameter BD is also angle bisector of $\angle ADC$.

It is given $BE = 5, ED = 15$. Find $BC = x$.



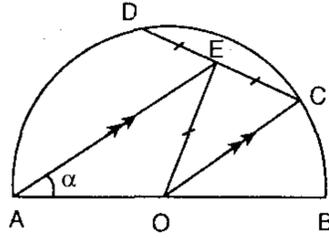
6. **(ZG-modified)** In the figure the right triangle ABC has sides $BC = 12, CA = 16$. The circle with center A and radius AC cuts AB again at M , and similarly the circle with center B and radius BC cuts AB again at K .

- (i) Find the area of the triangle KMC .
- (ii) Find $\sin \angle KCM$.

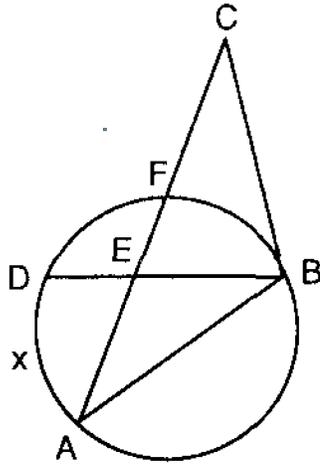


7. **(ZG)** In the figure O is the center, E is the midpoint of CD , and

$AE \parallel OC$. If $OE = EC$, find $\angle BAE = \alpha$.

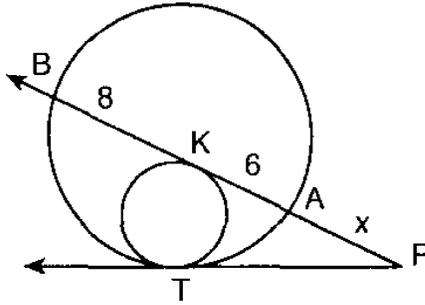


8. (ZG) In the figure $\widehat{DF} = 90$, $\widehat{FB} = 60$, and CB is tangent to the circle. If $\angle ACB = 50$, find $\widehat{AD} = x$.



9. (LAMC 2008) Let $ABCD$ be a quadrilateral inscribed in a circle with center O . Let M be the middle of the arc AB . Let $E = MC \cap AB$ and $K = MD \cap AB$. Show that the quadrilateral $KECD$ is cyclic.
10. (ZG) In the figure two circles are tangent to each other and line PT

at point T . If $BK = 8$, $KA = 6$, find $AP = x$.



11. $ABCD$ is cyclic quadrilateral with $AB = 1$, $BC = 2$, $CD = 3$, $DA = 4$. What is AC^2 ?
12. The circle with center O has diameter length $\sqrt{2425}$. Two chords AB and CD have midpoints M and N respectively. If $CD - AB = OM - ON = 2$, find the area of the triangle ODC .