Dad says that anyone who can’t use a slide rule is a cultural illiterate and should not be allowed to vote.

_Have Space Suit – Will Travel, 1958_

**Part 1: Logarithms**

**Definition 1:**
The logarithm is the inverse of the exponent. That is, if \( b^p = c \), then \( \log_b c = p \).
In other words, \( \log_b c \) asks the question “what power do I need to raise \( b \) to to get \( c \)?”

In both \( b^p \) and \( \log_b c \), the number \( b \) is called the base.

**Problem 1:**
Evaluate the following by hand:

1. \( \log_{10} (1000) \)
2. \( \log_2 (64) \)
3. \( \log_2 \left( \frac{1}{4} \right) \)
4. \( \log_x (x) \) for any \( x \)
5. \( \log_x (1) \) for any \( x \)
**Definition 2:**
There are a few ways to write logarithms:
\[
\begin{align*}
\log x &= \log_{10} x \\
\lg x &= \log_{10} x \\
\ln x &= \log_e x
\end{align*}
\]

**Definition 3:**
The *domain* of a function is the set of values it can take as inputs.
The *range* of a function is the set of values it can produce.

For example, the domain and range of \( f(x) = x \) is \( \mathbb{R} \), all real numbers.
The domain of \( f(x) = |x| \) is \( \mathbb{R} \), and its range is \( \mathbb{R}^+ \cap \{0\} \), all positive real numbers and 0.

Note that the domain and range of a function are not always equal.

**Problem 2:**
What is the domain of \( f(x) = 5^x \)?
What is the range of \( f(x) = 5^x \)?

**Problem 3:**
What is the domain of \( f(x) = \log x \)?
What is the range of \( f(x) = \log x \)?
Problem 4:
Prove the following identities:
1. $\log_b (b^x) = x$
2. $b^{\log_b x} = x$
3. $\log_b(xy) = \log_b(x) + \log_b(y)$
4. $\log_b \left( \frac{x}{y} \right) = \log_b(x) - \log_b(y)$
5. $\log_b(x^y) = y \log_b(x)$
Part 2: Introduction

Mathematicians, physicists, and engineers needed to quickly solve complex equations even before computers were invented.

The *slide rule* is an instrument that uses the logarithm to solve this problem. Before you continue, cut out and assemble your slide rule.

There are four scales on your slide rule, each labeled with a letter on the left side:

| Scale | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| T     |   |   |   |   |   |   |   |   |   | 15| 20 | 25 | 30 | 35 | 40 | 45 |   |   |   |
| K     | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10|   |   |   |   |   |   |   |   |   |   |
| A     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 |   |   |   |   |   |   |   |   |   |
| B     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 |   |   |   |   |   |   |   |   |   |
| CI    | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 1 |   |   |   |   |   |   |   |   |   |
| C     | 1 | 1.5|2|3|4|5|6|7|8|9|1   |   |   |   |   |   |   |   |   |   |
| D     | 1 | 1.5|2|3|4|5|6|7|8|9|1   |   |   |   |   |   |   |   |   |   |
| L     | 0 | .1|.2|.3|.4|.5|.6|.7|.8|.9|1   |   |   |   |   |   |   |   |   |   |   |
| S     | 6 | 7 | 8 | 9 | 10|15|20|25|30|40|50|60|90|

Each scale’s “generating function” is on the right:

- T: tan
- K: $x^3$
- A,B: $x^2$
- CI: $\frac{1}{x}$
- C, D: $x$
- L: log_{10}(x)
- S: sin

Once you understand the layout of your slide rule, move on to the next page.
Part 3: Multiplication

We’ll use the C and D scales of your slide rule to multiply. Say we want to multiply $2 \times 3$. First, move the left-hand index of the C scale over the smaller number, 2:

```
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>
```

Then we’ll find the second number, 3 on the C scale, and read the D scale under it:

```
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>
```

Of course, our answer is 6.

**Problem 5:**
What is $1.15 \times 2.1$?
Use your slide rule.

Note that your answer isn’t exact. $1.15 \times 2.1 = 2.415$, but an answer accurate within two decimal places is close enough for most practical applications.
Look at your C and D scales again. They contain every number between 1 and 10, but no more than that. What should we do if we want to calculate $32 \times 210$?

Problem 6:
Using your slide rule, calculate $32 \times 210$.

Problem 7:
Compute the following:
1. $1.44 \times 52$
2. $0.38 \times 1.24$
3. $\pi \times 2.35$
Problem 8:
Note that the numbers on your C and D scales are logarithmically spaced.

Why does our multiplication procedure work?
Now we want to compute 7.2 × 5.5:

No matter what order we go in, the answer ends up off the scale. There must be another way.

Look at the far right of your C scale. There’s an arrow pointing to the 10 tick, labeled right-hand index. Move it over the larger number, 7.2:

Now find the smaller number, 5.5, on the C scale, and read the D scale under it:

Our answer should be about 7 × 5 = 35, so let’s move the decimal point: 5.5 × 7.2 = 39.6. We can do this by hand to verify our answer.

Problem 9:
Why does this work?
Problem 10:
Compute the following using your slide rule:
1. $9 \times 8$
2. $15 \times 35$
3. $42.1 \times 7.65$
4. $6.5^2$
Part 4: Division

Now that you can multiply, division should be easy. All you need to do is work backwards. Let’s look at our first example again: $3 \times 2 = 6$.

We can easily see that $6 \div 3 = 2$

and that $6 \div 2 = 3$:

If your left-hand index is off the scale, read the right-hand one. Consider $42.25 \div 6.5 = 6.5$:

Place your decimal points carefully.
Problem 11:
Compute the following using your slide rule.
1. $135 \div 15$
2. $68.2 \div 0.575$
3. $(118 \times 0.51) \div 6.6$
Part 5: Squares, Cubes, and Roots

Now, take a look at scales A and B, and note the label on the right: $x^2$. If C, D are $x$, A and B are $x^2$, and K is $x^3$.

Finding squares of numbers up to ten is straightforward: just read the scale. Square roots are also easy: find your number on B and read its pair on C.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Problem 12:
Compute the following.
1. $1.5^2$
2. $3.1^2$
3. $7^3$
4. $\sqrt{14}$
5. $\sqrt[3]{150}$

Problem 13:
Compute the following.
1. $42^2$
2. $\sqrt{200}$
3. $\sqrt{2000}$
4. $\sqrt{0.9}$
5. $\sqrt[3]{0.12}$
Part 6: Inverses

Try finding $1 \div 32$ using your slide rule. The procedure we learned before doesn’t work!

This is why we have the CI scale, or the “C Inverse” scale.

Problem 14:
Figure out how the CI scale works and compute the following:

1. $\frac{1}{7}$
2. $\frac{1}{120}$
3. $\frac{1}{\pi}$
Part 7: Logarithms Base 10

When we take a logarithm, the resulting number has two parts: the *characteristic* and the *mantissa*. The characteristic is the integral (whole-numbered) part of the answer, and the mantissa is the fractional part (what comes after the decimal).

For example, \( \log_{10} 18 = 1.255 \), so in this case the characteristic is 1 and the mantissa is 0.255.

**Problem 15:**
Approximate the following logs without a slide rule. Find the exact characteristic, and approximate the mantissa.
1. \( \log_{10} 20 \)
2. \( \log_{2} 18 \)

Now, find the L scale on your slide rule. As you can see on the right, its generating function is \( \log_{10} x \).

**Problem 16:**
Compute the following logarithms using your slide rule. You’ll have to find the characteristic yourself, but your L scale will give you the mantissa. Don’t forget your log identities!
1. \( \log_{10} 20 \)
2. \( \log_{10} 15 \)
3. \( \log_{10} 150 \)
4. \( \log_{10} 0.024 \)
Part 8: Logarithms in Any Base

Our slide rule easily computes logarithms in base 10, but we can also use it to find logarithms in any base.

Proposition 1:
This is usually called the change-of-base formula:

\[ \log_b a = \frac{\log_c a}{\log_c b} \]

Problem 17:
Using log identities, prove Proposition 1.

Problem 18:
Approximate the following:
1. \( \log_2 56 \)
2. \( \log_{5.2} 26 \)
3. \( \log_{12} 500 \)
4. \( \log_{4.1} 134 \)
This page unintentionally left blank.
ASSEMBLY INSTRUCTIONS

1. Cut out the entire white panel (a). Cut along line between parts A and B (b), then remove excess (c).

2. Fold part A along the dotted lines.

3. Slip part B into the folded part A.