ORMC Olympiad Group
Fall: Week 3
Geometry: Similarity and Triangles

Osman Akar

October 10, 2022

Problems

1. **Ceva's Theorem** ABC is a triangle and P is an interior point. The cevians AP, BP, CP cuts the sides BC, CA, AB at points at A_1, B_1, C_1 respectively. Then
$$\frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} \cdot \frac{AC_1}{C_1B} = 1$$

2. Let ABC be a triangle with $BC = 70$ and points M and N are chosen on the sides AB and AC so that $MN \parallel BC$. Segments CM and BN intersect at the point K. A line which passes through K and parallel to BC intersects with the sides AB and AC at X and Y. Find MN if $XY = 42$.

3. **(TJNMO-FR 2017-modified)** Point E is chosen in a parallelogram $ABCD$ so that $\angle AEB + \angle DEC = 180°$. Prove that $\angle DAE = \angle DCE$

4. **(TNMO-FR 2018 - modified)** ABC is right triangle with hypotenuse AB and it is given that $AC/BC = 3/4$. The interior circle touches sides BC and AC at D and E respectively. AD intersects with the incircle again at the point S. Similarly BE intersects with the incircle again at T. BE and AD intersect at point K.

(a) Find AS/KD

(b) Find $(AS/TD)^2$

5. **(HMMT 2005 General)** A triangular piece of paper of area 1 is folded along a line parallel to one of the sides and pressed flat. What is the minimum possible area of the resulting figure?

6. **(Math Prize for Girls 2014)** Let ABC be a triangle. Points D, E, and F are respectively on the sides BC, CA, and AB of ABC. Suppose that
$$\frac{AE}{AC} = \frac{CD}{CB} = \frac{BF}{BA} = x$$
for some x with $\frac{1}{2} < x < 1$. Segments AD, BE, and CF cut the triangle into 7 nonoverlapping regions: 4 triangles and 3 quadrilaterals. The total area of the 4 triangles equals the total area of the 3 quadrilaterals. Compute the value of x. Express your answer in the form $\frac{k-\sqrt{m}}{n}$, where k and n are positive integers and m is a square-free positive integer.
Remark: Note that the figure is very similar to the figure in M.2

7. **(Prasolov 1.13)** In $\triangle ABC$ bisectors AA_1 and BB_1 are drawn. Prove that the distance from any point M of A_1B_1 to line AB is equal to the sum of distances from M to AC and BC.

8. $ABCD$ is a parallelogram, M is the midpoint of the side AD. Let H be the feet of the altitude from C to BM. Prove that $DH = CD$.

9. **(LAMC 2008)** Prove that in any triangle a median drawn to a side is smaller than half of the sum of the other two sides.

10. From a point A outside of the circle Γ the tangent AB is drawn, where B is the tangency point. Another line which passes through A cuts the circle Γ at points C and D. If $BC = 5, BD = 7$, what the maximum integer length that the segment AB can take?