ORMC Olympiad Group
Fall: Week 2
Geometry: Similarity and Triangles

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Problems

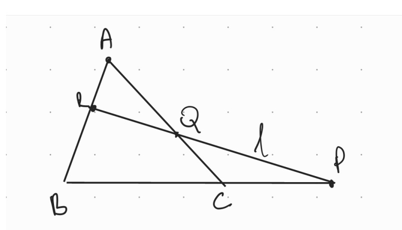
1. **(Prasolov 0.4)** On side BC of $\triangle ABC$ point A_1 is taken so that

$$BA_1 : A_1C = 2 : 1$$

What is the ratio in which median CC_1 divides segment AA_1?

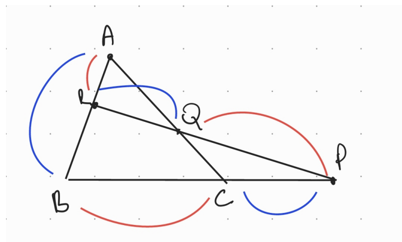
2. **(AIME 1986)** In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is then drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments are of an equal length d, find d.

3. **Menelaus' Theorem** ABC is a triangle. A line l cuts the segments AB and AC at R and Q, and cuts the extension of BC at P.



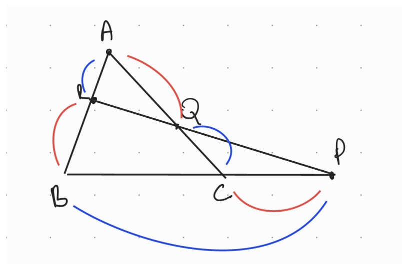
Then

$$\frac{AR}{AB} \cdot \frac{BC}{CP} \cdot \frac{PQ}{QR} = 1$$



and similarly

$$\frac{PC}{PB} \cdot \frac{BR}{RA} \cdot \frac{AQ}{QC} = 1$$



4. **Ceva's Theorem** ABC is a triangle and P is an interior point. The cevians AP, BP, CP cuts the sides BC, CA, AB at points at A_1, B_1, C_1 resppectively. Then
$$\frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} \cdot \frac{AC_1}{C_1B} = 1$$

5. Let ABC be a triangle with $BC = 70$ and points M and N are chosen on the sides AB and Ac so that $MN \parallel BC$. Segments CM and BN intersect at the point K. A line which passes through K and parallel to BC intersects with the sides AB and AC at X and Y. Find MN if $XY = 42$.

6. **(TJNMO-FR 2017-modified)** Point E is chosen in a parallelogram $ABCD$ so that $\angle AEB + \angle DEC = 180°$. Prove that $\angle DAE = \angle DCE$

7. **(TNMO-FR 2018 - modified)** ABC is right triangle with hypotenuse AB and it is given that $AC/BC = 3/4$. The interior circle touches sides BC and AC at D and E respectively. AD intersects with the incircle

again at the point S. Similarly BE intersects with the incircle again at T. BE and AD intersect at point K.

(a) Find AS/KD

(b) Find $(AS/TD)^2$

8. **(HMMT 2005 General)** A triangular piece of paper of area 1 is folded along a line parallel to one of the sides and pressed flat. What is the minimum possible area of the resulting figure?

9. **(Math Prize for Girls 2014)** Let ABC be a triangle. Points D, E, and F are respectively on the sides BC, CA, and AB of ABC. Suppose that
$$\frac{AE}{AC} = \frac{CD}{CB} = \frac{BF}{BA} = x$$
for some x with $\frac{1}{2} < x < 1$. Segments AD, BE, and CF cut the triangle into 7 nonoverlapping regions: 4 triangles and 3 quadrilaterals. The total area of the 4 triangles equals the total area of the 3 quadrilaterals. Compute the value of x. Express your answer in the form $\frac{k-\sqrt{m}}{n}$, where k and n are positive integers and m is a square-free positive integer. **Remark: Note that the figure is very similar to the figure in M.2**

10. **(Prasolov 1.13)** In $\triangle ABC$ bisectors AA_1 and BB_1 are drawn. Prove that the distance from any point M of A_1B_1 to line AB is equal to the sum of distances from M to AC and BC.