ORMC Olympiad Group
Fall: Week 1
Geometry: Similarity and Triangles

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Theorem 1 (**Basic Similarity and Congruence Rules**). *There are a number of ways to find similariry.*

1. *AA*
2. *SAS*
3. *SSS*

Problems

1. Let ABC be a triangle with $BC = 10, AC = 12$. Point E is chosen on the side AC so that $AE = 4$. Point K is chosen on the extension of the ray BE so that $AK \parallel BC$. What is AK?

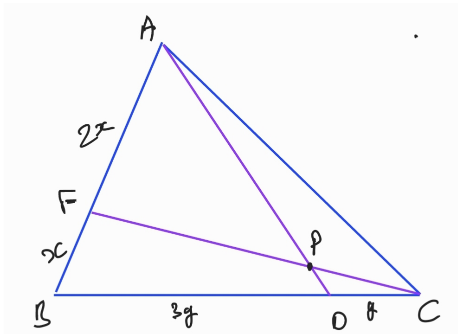
2. Let ABC be a triangle with $AB = 8, BC = 10, AC = 12$. Point E is chosen on the side AC so that $AE = 4$. Point K is chosen on the extension of the ray BE so that $AK \parallel BC$. Point F is chosen on the side AC so that $KF \parallel AB$. What is KF?

3. The point D is chosen on the side AC of triangle ABC. If $AB = 6$, $AD = 4$, $\angle BAC = 40$, $\angle ACB = 60$, $\angle DBC = 20$, find DC.

4. Points F and D are chosen on the sides AB and BC of triangle ABC. AD and CF intersect at the point P. It is given that $AB/BF = BD/DC = 3$.

    (a) what is AP/PD?

    (b) What is CP/PF?



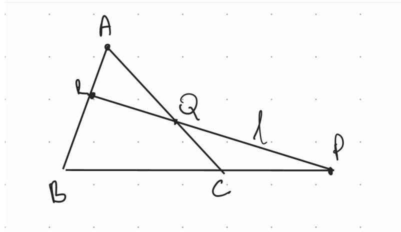
5. **(Prasolov 0.5)** Square $PQRS$ is inscribed into $\triangle ABC$ so that vertices P and Q lie on sides AB and AC and vertices R and S lie on BC. Express the length of the square's side in terms of a and h_a.

6. ABC is an equilateral triangle with side length 15. Points D, E, F are chosen on the sides BC, CA and AB respectively so that $BD = CE = AF = 7$. When we draw AD, BE, CF, we create a smaller equilateral triangle in the middle of ABC, say that $\triangle XYZ$. Side lenght of $\triangle XYZ$ can be represented as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

7. **(AIME 1986)** In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is then drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments are of an equal length d, find d.

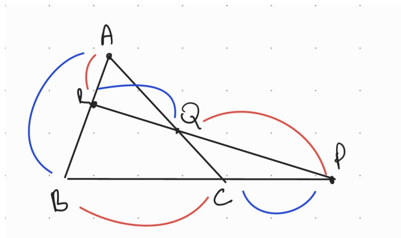
8. **Menelaus' Theorem** ABC is a triangle. A line l cuts the segments

AB and AC at R and Q, and cuts the extension of BC at P.



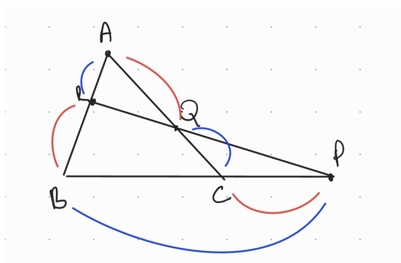
Then

$$\frac{AR}{AB} \cdot \frac{BC}{CP} \cdot \frac{PQ}{QR} = 1$$



and similarly

$$\frac{PC}{PB} \cdot \frac{BR}{RA} \cdot \frac{AQ}{QC} = 1$$



9. **Ceva's Theorem** ABC is a triangle and P is an interior point. The cevians AP, BP, CP cuts the sides BC, CA, AB at points at A_1, B_1, C_1 resppectively. Then

$$\frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} \cdot \frac{AC_1}{C_1B} = 1$$

10. Let ABC be a triangle with $BC = 70$ and points M and N are chosen on the sides AB and Ac so that $MN \parallel BC$. Segments CM and BN intersect at the point K. A line which passes through K and parallel

to BC intersects with the sides AB and AC at X and Y. Find MN if $XY = 42$.

11. **(TJNMO-FR 2017-modified)** Point E is chosen in a parallelogram $ABCD$ so that $\angle AEB + \angle DEC = 180°$. Prove that $\angle DAE = \angle DCE$

12. **(TNMO-FR 2018 - modified)** ABC is right triangle with hypotenuse AB and it is given that $AC/BC = 3/4$. The interior circle touches sides BC and AC at D and E respectively. AD intersects with the incircle again at the point S. Similarly BE intersects with the incircle again at T. BE and AD intersect at point K.

    (a) Find AS/KD

    (b) Find $(AS/TD)^2$