Slide Rules

Prepared by Mark on August 25, 2022

Dad says that anyone who can't use a slide rule is a cultural illiterate and should not be allowed to vote.

Have Space Suit – Will Travel, 1958

Part 1: Logarithms

Definition 1:
The *logarithm* is the inverse of the exponent. That is, if $b^p = c$, then $\log_b c = p$.
In other words, $\log_b c$ asks the question "what power do I need to raise b to to get c?"

In both b^p and $\log_b c$, the number b is called the *base*.

Problem 1:
Find the value of the following:

1. $\log_{10}(1000)$

2. $\log_2(64)$

3. $\log_2\left(\frac{1}{4}\right)$

4. $\log_x(x)$ for any x

5. $log_x(1)$ for any x

Definition 2:
There are a few ways to write logarithms:

$\log x = \log_{10} x$
$\lg x = \log_{10} x$
$\ln x = \log_e x$

Definition 3:
The *domain* of a function is the set of values it can take as inputs.
The *range* of a function is the set of values it can produce.

For example, both the domain of $f(x) = x$ are {all real numbers}.
The domain and range of $f(x) = \frac{1}{x}$ are $\{\text{all real numbers} \neq 0\}$.

Note that the domain and range of a function are not always equal.

Problem 2:
What is the domain of $f(x) = 5^x$?
What is the range of $f(x) = 5^x$?

Problem 3:
What is the domain of $f(x) = \log x$?
What is the range of $f(x) = \log x$?

Problem 4:
Prove the following identities:

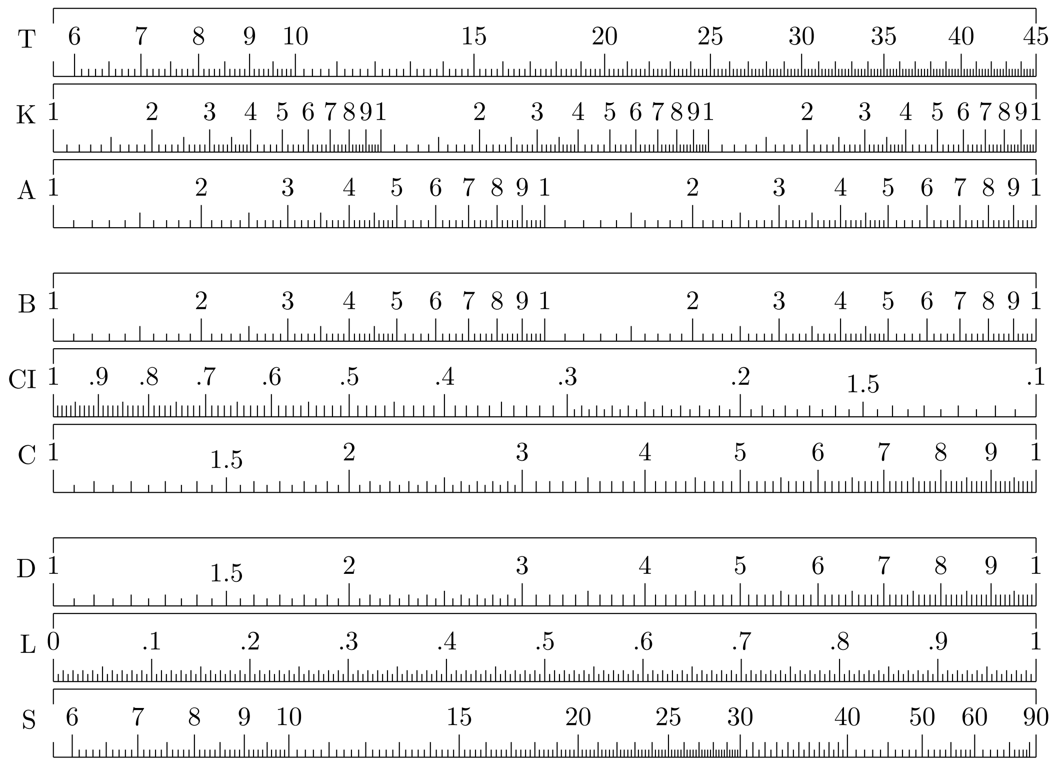
1. $\log_b(b^x) = x$
2. $b^{\log_b x} = x$
3. $\log_b(xy) = \log_b(x) + \log_b(y)$
4. $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
5. $\log_b(x^y) = y \log_b(x)$

Part 2: Introduction

Mathematicians, physicists, and engineers needed to quickly solve complex equations even before computers were invented.

The *slide rule* is an instrument that uses the power of logarithms to solve this problem. Before you continue, tear off the last page of this handout and assemble your slide rule.

There are four scales on your slide rule, each labeled with a letter on the left side:



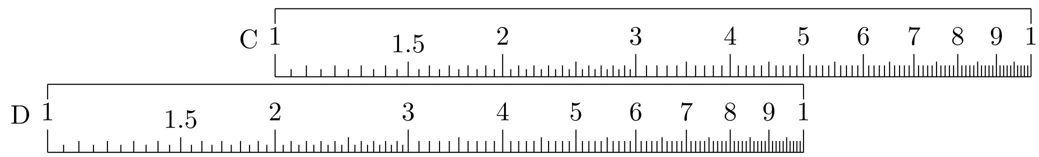
Each scale's "generating function" is on the right:

- T: tan
- K: x^3
- A,B: x^2
- CI: $\frac{1}{x}$
- C, D: x
- L: $\log_{10}(x)$
- S: sin

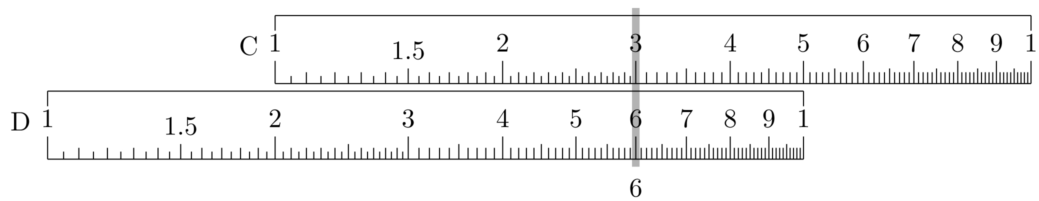
Once you understand the layout of your slide rule, move on to the next page.

Part 3: Multiplication

We'll use the C and D scales of your slide rule to multiply.
Say we want to multiply 2×3. First, move the *left-hand index* of the C scale over the smaller number, 2:



Then we'll find our second number, 3 on the C scale, and read the D scale under it:



Of course, our answer is 6.

Problem 5:
What is 1.15×2.1?
Use your slide rule.

Note that your answer isn't exact. While $1.15 \times 2.1 = 2.415$, an answer accurate within two decimal places is close enough for most practical applications.

Now, look at your C and D scales again. They contain every number between 1 and 10, but no more than that. What should we do if we want to calculate 32×210?

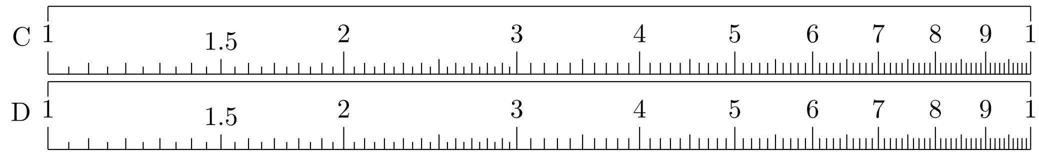
Problem 6:
Using your slide rule, calculate 32×210.
Hint: $32 = 3.2 \times 10^1$

This method of writing numbers is called *scientific notation*. In the form $a \times 10^b$, a is called the *mantissa*, and b, the *exponent*.
You may also see expressions like 4.3e2. This is equivalent to 4.3×10^2, but is more compact.

Problem 7:
Calculate the following:

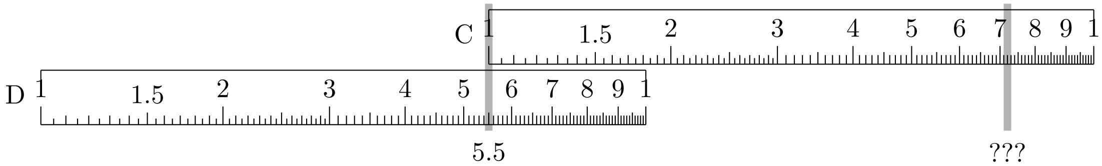
1. 1.44×52
2. 0.38×1.24
3. $\pi \times 2.35$

Problem 8:
Note that the numbers on your C and D scales are logarithmically spaced.



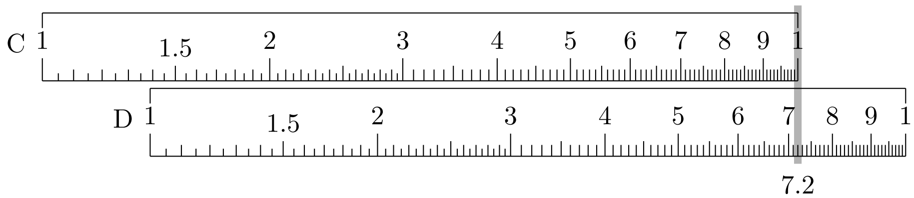
Why does our multiplication procedure work?
Hint: See Problem 4

Now we want to compute 7.2×5.5:

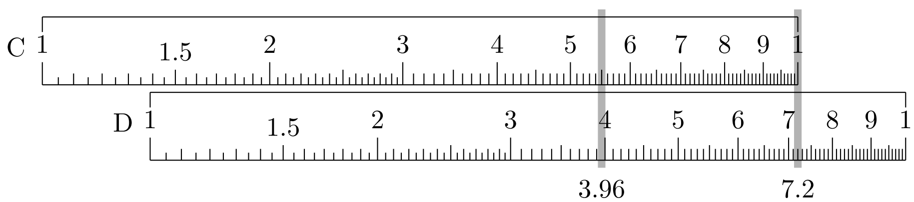


No matter what order we go in, the answer ends up off the scale. There must be another way.

Look at the far right of your C scale. There's an arrow pointing to the 10 tick, labeled *right-hand index*. Move it over the *larger* number, 7.2:



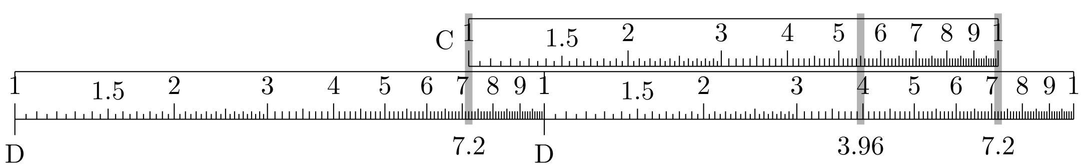
Now, find the smaller number, 5.5, on the C scale, and read the D scale under it:



Our answer should be about $7 \times 5 = 35$, so let's move the decimal point: $5.5 \times 7.2 = 39.6$. We can use a calculator (or long-multiplication) to verify our answer.

Why does this work?

Consider the following picture, where I've put two D scales next to each other:



The second D scale has been moved to the right by $(\log 10)$, so every value on it is $(\log 10)$ smaller than it should be.

In other words, the answer we get from reverse multiplication is the following: $\log a + \log b - \log 10$. This reduces to $\log\left(\frac{a \times b}{10}\right)$, which explains the misplaced decimal point in 7.2×5.5.

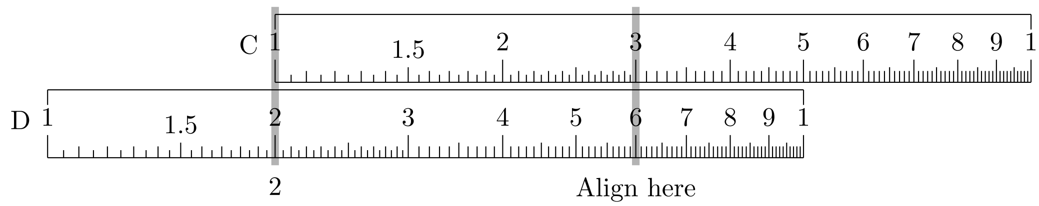
Problem 9:
Compute the following using your slide rule:

1. 9×8
2. 15×35
3. 42.1×7.65
4. 6.5^2

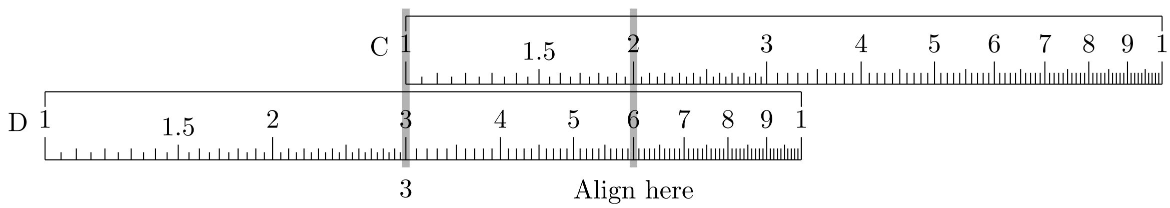
Part 4: Division

Now that you can multiply, division should be easy. All you need to do is work backwards. Let's look at our first example again: $3 \times 2 = 6$.

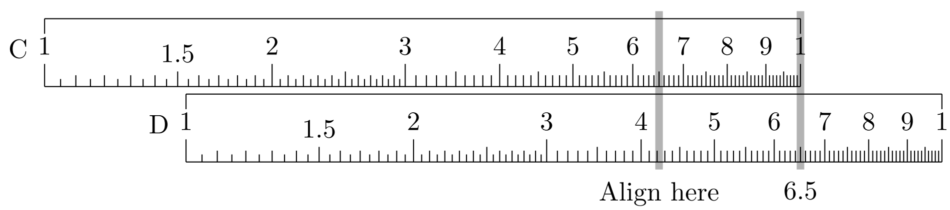
We can easily see that $6 \div 3 = 2$



and that $6 \div 2 = 3$:



If your left-hand index is off the scale, read the right-hand one.
Consider $42.25 \div 6.5 = 6.5$:



Place your decimal points carefully.

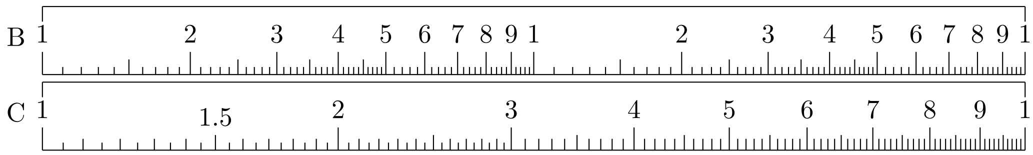
Problem 10:
Compute the following using your slide rule.

1. $135 \div 15$
2. $68.2 \div 0.575$
3. $(118 \times 0.51) \div 6.6$

Part 5: Squares, Cubes, and Roots

Now, take a look at scales A and B, and note the label on the right: x^2. If C, D are x, A and B are x^2, and K is x^3.

Finding squares of numbers up to ten is straightforward: just read the scale.
Square roots are also easy: find your number on B and read its match on C.



Problem 11:
Compute the following.

1. 1.5^2
2. 3.1^2
3. 7^3
4. $\sqrt{14}$
5. $\sqrt[3]{150}$

Problem 12:
Compute the following.

1. 42^2
2. $\sqrt{200}$
3. $\sqrt{2000}$
4. $\sqrt{0.9}$
5. $\sqrt[3]{0.12}$

Part 6: Inverses

Try finding $1 \div 32$ using your slide rule.
The procedure we learned before doesn't work!

This is why we have the CI scale, or the "C Inverse" scale.

Problem 13:
Figure out how the CI scale works and compute the following:

1. $\frac{1}{7}$
2. $\frac{1}{120}$
3. $\frac{1}{\pi}$

Part 7: Logarithms Base 10

When we take a logarithm, the resulting number has two parts: the *characteristic* and the *mantissa*. The characteristic is the integral (whole-numbered) part of the answer, and the mantissa is the fractional part (what comes after the decimal).

For example, $\log_{10} 18 = 1.255$, so in this case the characteristic is 1 and the mantissa is 0.255.

Problem 14:
Approximate the following logs without a slide rule. Find the exact characteristic, and approximate the mantissa.

1. $\log_{10} 20$
2. $\log_{2} 18$

Now, find the L scale on your slide rule. As you can see on the right, its generating function is $\log_{10} x$.

Problem 15:
Compute the following logarithms using your slide rule.
You'll have to find the characteristic yourself, but your L scale will give you the mantissa.
Don't forget your log identities!

1. $\log_{10} 20$
2. $\log_{10} 15$
3. $\log_{10} 150$
4. $\log_{10} 0.024$

Part 8: Logarithms in Any Base

Our slide rule easily find logarithms base 10, but we can also use it to find logarithms in *any* base.

Proposition 1:
This is usually called the *change-of-base* formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Problem 16:
Using log identities, prove Proposition 1.

Problem 17:
Approximate the following:

1. $\log_2 56$
2. $\log_{5.2} 26$
3. $\log_{12} 500$
4. $\log_{43} 134$

a Cut here

c Cut here

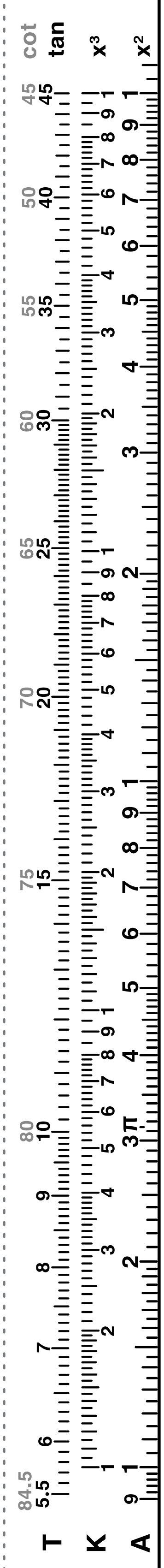
ASSEMBLY INSTRUCTIONS

1 Cut out the entire white panel (*a*). Cut along line between parts A and B (*b*), then remove excess (*c*).

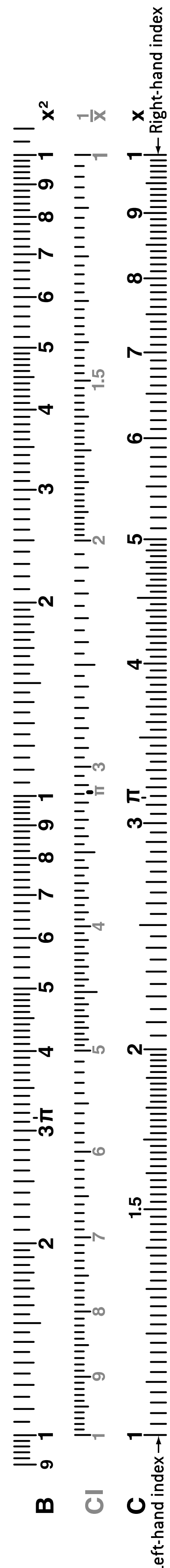
2 Fold part A along the dotted lines.

3 Slip part B into the folded part A.

Part **A**



b Cut here



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Part **B**