# Week 7: Word Problems

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**Problem 1.** *A woodsman starts the day with a pile of logs, and he saws those logs as well as trees to create new logs. After making 52 cuts he sees there are 72 logs. How many logs did he start the day with?*

**Problem 2.** *A chessboard has matches placed at all of the edges of its squares (one match per edge). A rook begins in the corner of the board, and it can move vertically and horizontally (in the normal way for a rook), but it can't move through any edge that has a match on it. What is the smallest number of matches you can remove from the board so that the rook may visit every square?*

**Problem 3.** *There are three piles, each with 75 matches. Alan, Becky, Carl, and Dana play a game. They take turns: at each turn, the player must split any existing pile into two new piles, dividing up the matches any way they like, but each new pile must have at least one match. Alan goes first and divides the first pile into one pile with 40 matches and one pile with 35. Becky goes next, then Carl, then Dana. The winner is the last person able to make a valid move (split a pile). Who wins the game?*

**Problem 4.** *A king has three sons. 100 of his descendants had 3 sons, and the rest died without having children. How many descendants does the king have?*

**Problem 5.** *There is a tree growing in Arborville whose main trunk splits into six branches. Of all the branches on the tree, 10 split into two branches, 20 split into three branches, and 30 split into four branches; the rest of them dead end and grow 5 leaves. How many branches does the tree have? How many leaves?*

**Problem 6.** *There are 30 trees in Saplingrad strong enough to hold up the end of a hammock. Every such tree has two hammocks attached to it. Prove that there is a loop of trees connected by hammocks.*

**Problem 7.** *Vera, Wei, Xavier, Yvette, and Zach exchanged handshakes (one handshake for every pair of people). How many handshakes do we have in all?*

**Problem 8.** *Treeland consists of 2,010 cities in the woods. There are paths between cities, but no two paths ever meet (except at their endpoints, when they both end at the same city). It is possible to travel from any city to any other city by going along the paths from city to city, but there is only one way to do so for any pair of starting city and destination city. How many paths are there in Treeland?*

**Problem 9.** *Trans Wald Airlines started out as a small company flying planes between two cities. When business gets better they add flights between one of their current cities and a new city they haven't flown to before. Prove that there is only one way to get from one city to another using Trans Wald.*

**Problem 10.** *A volleyball net has a grid that is 50 squares by 400 squares. How many segments of the net can you cut without cutting the net into two pieces?*

**Problem 11.** *There are 20 problems being solved in Math Circle and presented at the board. Every student solved exactly two problems, and every problem was solved by exactly two students. Show that you can organize the presentation of solutions in such a way that every student would present one of the problems they solved.*

**Problem 12.** *Sarah and David play a game. They have a piece of chocolate with 50 squares in it: 5 rows and 10 columns. Sarah breaks off the first two columns (10 squares). David breaks off the top row (2 squares) of that piece. They continue taking turns breaking the chocolate; at each turn they choose a piece, and break off some number of rows or columns. The winner is the last person able to make a break (and gets to eat all the chocolate). Does one of them have a winning strategy? Who?*