

PROOF, LOGIC, SYLLOGISM

MATH CIRCLE (BEGINNERS) 04/22/2012

(1) **Syllogisms** A syllogism is a collection of logical premises and a conclusion that may be deduced from them. It is a very simple kind of proof. A famous example is:

- All men are mortal (*first premise*)
- Socrates is a man. (*second premise*)
- Therefore, Socrates is mortal (*conclusion*)

Not all syllogisms have exactly this form, but they all employ similar reasoning. Here are some fanciful examples from Lewis Carroll of *Alice in Wonderland* fame, who also happened to be an enthusiast and creator of logic puzzles. In each case you must discover a logical conclusion which uses all three premises (these examples, unlike that above, use THREE premises, which are listed, and have a conclusion which is NOT written—you have to find it!)

(a) Syllogism #1

- Babies are illogical.
- Nobody is despised who can manage a crocodile.
- Illogical persons are despised.

(b) Syllogism #2

- My saucepans are the only things that I have that are made of tin.
- I find all your presents very useful.
- None of my saucepans are of the slightest use.

(c) Syllogism #3

- No potatoes of mine, that are new, have been boiled.
- All of my potatoes in this dish are fit to eat.
- No unboiled potatoes of mine are fit to eat.

(d) Syllogism #4

- No ducks waltz.
- No officers ever decline to waltz.
- All my poultry are ducks.

(2) *You Be The Syllogist* Design your own syllogism in the style of the ones above. First do one with just two premises, like the Socrates example. Then try doing one with three different premises, like Lewis Carroll's examples. When you've got it, write the premises down on a separate sheet and trade with someone to try to solve each other's.

(3) *Who Has the Beard?** "I've only met one of them," said Martha. "The guy with the beard. Which is he?"

"You figure it out yourself," Clem replied. "Two of them are married, two have blue eyes, and two are clean-shaven. The bearded one has brown eyes. Doug's wife is Ken's sister, and the bachelor has the same color eyes as Joe. They're three great guys."

Who had the beard?

Who Has The Beard? Fill-in-the-Blanks

The men are Joe, Doug, and _____ (Ken, John, Peter, Doug), and Doug is _____ (clean-shaven, bachelor, married, blue-eyed). The bachelor is not Joe, so _____ (Ken, Doug) is the bachelor and he has the same color eyes as _____ (Ken, Doug, Joe). Hence Ken and Joe have _____ (brown, blue, hazel) eyes. Then Doug has brown eyes and is _____ (bachelor, clean-shaven, married, bearded).

*A. Bogomolny, *Who Has the Beard?* an outline solution to a simple word problem from *Interactive Mathematics Miscellany and Puzzles*

<http://www.cut-the-knot.org/Outline/Logic/WhoHasBeard.shtml>, Accessed 22 April 2012

(4) *Diamond Thief #1** Scheherazade began: "It has reached me, O Auspicious King, that one of Ali Baba's famous forty thieves stole into Abdul's shop and robbed him of some diamonds. Fortunately, they were all recovered, and then it was determined that the thief was either Sabit, Salim, or Shamhir - all of whom were in Ali Baba's famous robber band. At the trial, each of the three accused one of the others, but Shamhir is the only one who lied. Is he necessarily guilty?"

"Not necessarily," replied the king. "An innocent man might lie in order to protect his friend."

Is Shamhir necessarily guilty?

Diamond Thief #1 Fill-in-the-Blanks

The mere fact that _____ (Sabit, Salim, Shamhir) lied does not prove his guilt, but the additional fact that the other two _____ (lied, told the truth) does, because since Sabit truthfully accused one of the others, he must be _____ (guilty, innocent, hungry, blond) and the same with _____ (Sabit, Salim, Shamhir). Therefore, _____ (Sabit, Salim, Shamhir) is indeed the guilty one.

*A. Bogomolny, Robbery #1: an outline solution to a simple word problem from Interactive Mathematics Miscellany and Puzzles

<http://www.cut-the-knot.org/Outline/Logic/AliBabaRobbery1.shtml>, Accessed 22 April 2012

(5) **Diamond Thief #2*** Abdul's shop was robbed once more, but the loot was recovered. Again, there were three suspects; their names were Abu, Ibn, and Hasib. At the trial, they made the following statements:

- Abu: I did not commit the robbery!
- Ibn: Hasib certainly didn't!
- Hasib: Yes, I did!

Later on, two of them confessed to having lied. Who committed the robbery?

Diamond Thief #2 Fill-in-the-Blanks

This time, I won't give you the options to put in the blanks—you have to decide what should go in each one!

Since Ibn and _____ contradict each other, one of them lied and the other _____ . But since _____ of them lied, then _____ must be one of them. But he claims innocence. Hence Abu is _____ .

A. Bogomolny, Robbery #2: an outline solution to a simple word problem from Interactive Mathematics Miscellany and Puzzles

<http://www.cut-the-knot.org/Outline/Logic/AliBabaRobbery2.shtml>, Accessed 22 April 2012

(6) *Diamond Thief #3**

"Soon after, there was another robbery," said Scheherazade, and the same suspects - Abu, Ibn, and Hasib - were put on trial. They made the following statements:

Ibn: Hasib never committed the robbery! Hasib: That is true! Abu: Ibn is innocent!
 "Curiously enough, the actual thief told the truth, but they did not all tell the truth.
 Which one committed the robbery?"

Diamond Thief #3: You Make the Blanks!

This time there is no fill-in-the-blanks already—instead, you have to write one yourself. When you finish, copy to a new page and exchange with a partner to see if they can fill in your blanks.

(Don't put too many blanks in! You need to leave some names and important phrases so the other person can follow what you're saying. For instance, the following is not good (adapted from Diamond Thief #2):

An example of too many blanks (not for solving): Since _____ and _____ contradict each other, one of them _____ and the other _____. But since _____ of them _____, then _____ must be one of them. But he claims _____. Hence _____ is _____.)

*A. Bogomolny, Robbery #3: an outline solution to a simple word problem: from Interactive Mathematics Miscellany and Puzzles

<http://www.cut-the-knot.org/Outline/Logic/AliBabaRobbery3.shtml>, Accessed 22 April 2012

(7) *Diamond Thief #4**

Again, Abu, Ibn, and Hasib were on trial and it was known that one and only one of them was guilty. Abu claimed to be innocent; Ibn agreed that Abu was innocent; and Hasib claimed that he himself was the guilty one. As it turned out, the guilty one lied. Which one was guilty?

Diamond Thief #4: You Make the Blanks!

Same as #3—solve the puzzle, write a paragraph (with blanks), and exchange with a partner.

A. Bogomolny, Robbery #4: an outline solution to a simple word problem from Interactive Mathematics Miscellany and Puzzles

<http://www.cut-the-knot.org/Outline/Logic/AliBabaRobbery4.shtml>, Accessed 22 April 2012

(4) ***Magic Paint Buckets*** I have several buckets of magical paint (including colors such as *Green-Onion Red*, *Existentialist Yellow*, and *Pajama-Pants Blue*). I can take any two paint colors, mix them together, and get a new paint color (which might actually be the same as one of the original colors). No matter how I pick two colors and mix them together, I always get a color that I already owned.

There is a very special color called ***Crystal Clear*** which has the property that no matter what color I mix it with, I always get that color back as the result. (So *Crystal Clear* + *Existentialist Yellow* = *Existentialist Yellow*, and *Pajama-Pants Blue* + *Crystal Clear* = *Pajama-Pants Blue*, and *Crystal Clear* + *Crystal Clear* = *Crystal Clear*).

Every color X has at least one other color Y which is an **anti-color** for X. If I mix two anti-colors together, I get *Crystal Clear* as the resulting color.

Oh, and one more thing I should mention. One of the magical properties of this paint is that it's possible to mix a color with itself and get a *different* color as the result (weird, right?). Nevertheless, color mixes don't change: If *Green-Onion Red* + *Cow Purple* = *Magic Magenta*, then it will forever and always be the case that *Green-Onion Red* + *Cow Purple* = *Magic Magenta*. This holds true for mixing colors in different amounts, and different orders, also. For instance if I mix *Red* with *Green*, and separately mix *Green* with *Purple*, and then mix the two resulting colors together, the final result is the same as if I were to mix *Red* with *Purple*, and separately mix *Green* with *Green*, and then mix those results together. (Because in both cases I ultimately mixed 1 part *Red*, 1 part *Purple*, and 2 parts *Green*).

(a) My friend tells me that he took a bucket of my paint labeled *Shmorange Orange* and mixed it with paint from a second bucket whose label was worn off. The resulting color was still *Shmorange Orange*. Your friend says he concluded that the second bucket must have been *Crystal Clear* paint. Is he correct? (Note: It certainly **could have been** *Crystal Clear* paint. The question is, is it **possible** there is another color that mixes with *Shmorange Orange* and gives back *Shmorange Orange* as the result?) (Hint: What happens if you mix in an anti-color for *Shmorange Orange*?)

(b) We know that each color has *at least one* anti-color. Prove that each color has *exactly one* anti-color. (Hint: Start with an arbitrary color X. What if Y is an anti-color, and Z is also an anti-color?)

(c) Suppose I had exactly three colors—*Crystal Clear*, *Existentialist Yellow*, and *Pajama-Pants Blue*. (Clear, Yellow, and Blue for short, or C, Y, and B for really short). Figure out what the following mixes give you:

Clear + Clear =

Clear + Yellow =

Clear + Blue =

Yellow + Yellow =

Yellow + Blue =

Blue + Blue =