Topics in Plane Curves

Conic Sections

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. By making a change of variables, we can always transform such a curve to one of the standard forms for an ellipse/circle, parabola, or hyperbola.

1. Classify each of the following conic curves:
   (a) $xy = 1$ (b) $x^2 + xy + y^2 = 1$ (c) $x^2 + 2xy - y^2 = 0$

Intersection Number

Properties

1. $I_O(f, g)$ is a nonnegative integer or ∞.
2. $I_O(f, g) = I_O(g, f)$
3. $I_O(f, g) \geq 1$ if and only if f and g both contain the origin.
4. $I_O(x, y) = 1$
5. $I_O(f, g) = I_O(f, g + fh)$
6. $I_O(f, gh) = I_O(f, g) + I_O(f, h)$

Find the intersection number at the origin for each of the following pairs of curves:

1. $y = x^2$ and $y = 0$
2. $y = x^2$ and $y = x$
3. $y = 2x + 1$ and $y = x$
4. $y = x^2$ and $y^3 + 2xy + x^6 = 0$
5. $y^3 = x^2$ and $y^2 = x^3$
6. $y = x^3$ and $y^4 + 6x^3y + x^8 = 0$
7. $y = x^2 - 2x$ and $y^2 + 5y = 4x^3$
8. $y = x^2 + x$ and $y^2 + 5y = 4x^3$