Pigeonhole Principle

Problem 1 *Is it possible to cover an equilateral triangle with two smaller equilateral triangles? Why or why not?*

Problem 2 *You are given $n+1$ integers. Prove that there exist at least two of them such that their difference is divisible by n.*

Problem 3 *You are given a (8×8) chess board with a pair of opposite corner squares cut off. You are further given a set of dominoes each equal in size to a pair of the board squares with a common side. Is it possible to tile the board with the dominoes in such a way that all the board squares are covered while the dominoes neither overlap nor stick out?*

Problem 4 *The ocean covers more than a half of the Earth's surface. Prove that the ocean has at least one pair of antipodal points.*

Problem 5 *There are $n > 1$ people at a party. Prove that among them there are at least two people who have the same number of acquaintances at the gathering. (We assume that if A knows B, then B also knows A)*

Problem 6 *Among any five points with integer coordinates in the plane, there exist two such that the center of the line segment that connects them has integer coordinates as well.*

Problem 7 *Prove that if every point on a straight line is painted either black or white, then there exist three points of the same color such that one is the midpoint of the line segment formed by the other two.*

Problem 8 *All the points in the plain are painted with either one of two colors. Prove that there exist two points in the plain that have the same color and are located exactly one foot away from each other.*

Problem 9 *Each point of a circumference is colored either black or white. Prove that there exist three equally spaced points of the same color.*

Problem 10 *Let n be an integer not divisible by 2 and 5. Show that n has a multiple consisting entirely of ones.*

Problem 11 *Prove that for any $n > 1$, there exists an integer made of only sevens and zeros that is divisible by n.*

Problem 12 *One chooses $n+1$ numbers between 1 and $2n$. Show that she/he has selected two co-prime numbers.*

Problem 13 *One chooses $n+1$ numbers between 1 and $2n$. Show that she/he has selected two numbers a and b such that a divides b.*

Problem 14 *Prove that it is always possible to choose a subset of the set of integral numbers $a_1, a_2, \ldots, a_n$ so that the sum of the numbers in the subset is divisible by n.*

Problem 15 *Prove that there exist a positive integer divisible by 2013 such that its last four digits are 2014.*

Problem 16 *Let n be an odd number. Let $a_1, a_2, \ldots, a_n$ be a permutation of the numbers $1, 2, \ldots, n$. Prove that the product $(a_1-1)\times(a_2-2)\times\ldots\times(a_n-n)$ is an even number.*

Problem 17 *A stressed-out UCLA student consumes at least one espresso every day of a particular year, drinking 500 overall. Prove that on some consecutive sequence of whole days the student drinks exactly 100 espressos.*

Problem 18 *Prove that at a party with ten or more people, there are either three mutual acquaintances or four mutual strangers.*

Problem 19 *Given a table with a marked point, O, and with 2013 properly working watches put down on the table, prove that there exists a moment in time when the sum of the distances from O to the watches' centers is less than the sum of the distances from O to the tips of the watches' minute hands.*