Dr. Oleg Gleizer
prof1140g@math.ucla.edu

Dr. Olga Radko

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12 The *or* and negation operations

Warm-up

Once upon a time in a land far, far away there lived a very beautiful princess, the only daughter of a very evil king. Some day, a handsome and very smart prince from a neighboring kingdom came to pay them a visit. The princess and prince fell in love with each other and asked the king for a permission to marry. The evil king didn't want his daughter to leave. Instead of blessing the marriage, he ordered to put the prince in jail and to prepare for his execution.

The princess begged the king not to kill the prince and finally he agreed. He told the prisoner, "At her Highness's request, I will give you a chance. Tomorrow you will be brought to my court. You will have to pull a lot. I will put two pieces of paper in the box. One will read LIFE, the other will read DEATH. Whichever you pull out, it will be your destiny."

The king was a very evil man. He ordered his minister of justice to write DEATH on both notes. The princess overheard the king's order and found a way to warn the prince.

Problem 12.1 *What should the prince do to survive? Hint: kings do not like public embarrassment.*

12.1 Lesson

The truth function and the *or* operation.

Suppose the logic operation *or* is used to make a composite statement out of two other statements. Let us reuse the statements from problem **??** to figure out how the truthfulness of the composite statement depends on the truthfulness of the parts. Recall that a statement s being true is equivalent to $T(s) = 1$. A statement s being false is equivalent to $T(s) = 0$.

- $s =$ `Sky is blue.` $T(s) =$ 1
- $s =$ `Sky is green.` $T(s) =$ 0
- $s =$ `Grass is pink.` $T(s) =$ 0
- $s =$ `Grass is green.` $T(s) =$ 1

Statements in the next problem are composed of the statements above with the help of the operation *or*.

Problem 12.2 *Find the values of the truth function for the statements below.*

- $s =$ `Sky is green or grass is pink.` $T(s) =$ _____
- $s =$ `Sky is green or grass is green.` $T(s) =$ _____
- $s =$ `Sky is blue or grass is pink.` $T(s) =$ _____
- $s =$ `Sky is blue or grass is green.` $T(s) =$ _____

Out of the four composite statements considered on the previous page, the truth function takes the value 0 just once. This happens when both statements making up the composite statement are false. We see an important law of logic at work:

Let a composite statement be made of two other statements with the help of the logic operation *or*. Then the composite statement is false only if each of the statements that make it up is false. In all other cases, the composite statement is true.

$T(s_1)$	$T(s_2)$	$T(s_1 \text{ or } s_2)$
0	0	0
1	0	1
0	1	1
1	1	1

(12.1)

Problem 12.3 *A knight from the Island of Knights and Liars makes a statement s_1. A liar from the island makes a statement s_2. A tourist puts the statements together into a new statement: s_1 or s_2. Find the value of the truth function on the latter statement.*

Answer: $T(s_1 \text{ or } s_2) =$ _____

Problem 12.4 *A liar from the Island of Knights and Liars makes a statement s_1. Another liar makes a statement s_2. A tourist combines the statements together into a new statement: s_1 or s_2. Find the value of the truth function on the latter statement.*

Answer: $T(s_1 \text{ or } s_2) =$ _____

Negation

Negation is a function that takes any statement s as an input and produces the opposite statement $\bar{s}$ as the output. For example, let

$$s = \texttt{A knight always tells the truth.}$$

Then

$$\bar{s} = \texttt{A knight does not always tell the truth.}$$

Less formally, to negate a statement means to say, "No, no, no, it's the exact opposite of what you just said!" To practice, let us negate a few statements we have seen before.

Problem 12.5

- $s = \texttt{A mobster is a big lobster.}$

  $\bar{s} =$ _____

- $s = \texttt{The word } \textit{polygon} \texttt{ means a } \textit{3D} \texttt{ solid in Ancient Greek.}$

  $\bar{s} =$ _____

  _____

- $s = \texttt{Children went to see a movie.}$

  $\bar{s} =$ _____

Let $\bar{s}$ be the negation of a statement s. Since $\bar{s}$ claims the exact opposite of what s claims, the truth function must take different values on s and $\bar{s}$. If s is true, then $\bar{s}$ must be false and the other way around. For example, consider

$$s = \texttt{A mobster is a big lobster.} \text{ and}$$

$$\bar{s} = \texttt{A mobster is not a big lobster.}$$

In this case, $T(s) = 0$ and $T(\bar{s}) = 1$. Let us summarize with a negation truth table.

$T(s)$	$T(\bar{s})$
0	1
1	0

(12.2)

Problem 12.6 *Find $T(\bar{s})$ for the statements s given below.*

- $s = \texttt{Sky is blue.}$ $T(\bar{s}) =$ _____
- $s = \texttt{Sky is green.}$ $T(\bar{s}) =$ _____
- $s = \texttt{Grass is pink.}$ $T(\bar{s}) =$ _____
- $s = \texttt{Grass is green.}$ $T(\bar{s}) =$ _____
- $s = \texttt{Sky is blue and grass is pink.}$ $T(\bar{s}) =$ _____
- $s = \texttt{Sky is blue or grass is pink.}$ $T(\bar{s}) =$ _____

Problem 12.7 *A knight from the Island of Knights and Liars makes a statement s_1. A liar from the island makes a statement s_2. A tourist combines both statements into a new statement*

$$s = s_1 \text{ and } s_2$$

and then negates it. Is the negated statement true or false? Circle the correct answer. Explain your choice.

True *False*

Problem 12.8 *Alice, Bob, and Charlie live on the Island of Knights and Liars.*

- *Alice says that all three of them are liars.*
- *Bob agrees with her.*
- *Charlie says that only two of them are liars.*

What kind are they?

Answer: Alice is a _____ ,

Bob is a _____ ,

Charlie is a _____ .

Problem 12.9 *One year, there were exactly four Tuesdays and Saturdays in July. What day was July 31st?*

Answer: _____

12.2 Homework

Finish solving all the problems from class.

Tell your parents the story about a handsome prince, a beautiful princess, and an evil king from pages 1 and 2. Then explain how logic helped the prince to survive.

Problem 12.10 *The problem continues to the next page.*

- *Write down the statements s_1 and s_2 making up the composite statement s below.*

$s =$ `Jupiter or Earth is the largest planet of the Solar system.`

$s_1 =$ _____

_____

$s_2 =$ _____

_____

- *Write down the formula for s in terms of s_1, s_2, and a proper logic operation.*

$$s = \_\_\_\_\_$$

- *Find the values the truth function takes on the statements s_1 and s_2.*

$$T(s_1) = \_\_\_\_\_ \qquad T(s_2) = \_\_\_\_\_$$

- *Find the values the truth function takes on the statements s and $\bar{s}$.*

$$T(s) = \_\_\_\_\_ \qquad T(\bar{s}) = \_\_\_\_\_$$

Problem 12.11 *A liar from the Island of Knights and Liars makes a statement s_1. Another liar makes a statement s_2. A tourist combines both statements into a new statement*

$$s = s_1 \text{ or } s_2$$

and then negates it. Is the negated statement true or false? Circle the correct answer. Explain your choice.

True *False*

Let us continue the story about a handsome prince, a beautiful princess, an evil king, and the usefulness of logic.

The father of the prince, the king of the neighboring country, had his peculiarities, too. The old man invented the following way of punishing criminals. Law-breakers were given a choice between two doors. Behind each door, there was either a hungry tiger or a treasure of gold, but not nothing or both. The king would also post some warnings on the doors and then let the criminals choose.

Problem 12.12 *One day, there was a criminal facing the doors with the following signs.*

Door 1: at least one of these rooms has gold.

Door 2: a tiger is in the other room.

"Are the signs true?" asked the prisoner. "They are either both true or both false," replied the king. Then he smiled warmly and added, "Make your choice, buddy!"

Which door should the prisoner open? Why?

Answer: _____