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11 The truth function and the *and* operation

Warm-up

Problem 11.1 *It is raining at midnight on Tuesday. Do you think we can expect sunny weather in 48 hours? Circle the correct answer. Explain your choice.*

Yes *No* *Hard to tell*

11.1 Lesson

A *statement* is a sentence which is either *True* or *False*. For example, *I am a knight.* is a statement. *Are you a liar?* is not.

Problem 11.2 *Decide which sentences below are statements and which are not. Circle correct answers. Explain your choices. The problem continues to the next page.*

- *A knight always tells the truth.*

Statement *Not statement*

- *A liar always tells the truth.*

Statement *Not statement*

- *Does a tourist always tell the truth?*

Statement *Not statement*

- *Smoking is forbidden on UCLA campus.*

Statement *Not statement*

- *Please refrain from smoking on board of the aircraft.*

Statement *Not statement*

- $11 + 1 = 100$

Statement *Not statement*

- *This statement is a lie.*

Statement *Not statement*

Problem 11.3 *Use the lines below to write a sentence that is a statement.*

Problem 11.4 *Use the lines below to write a sentence that is not a statement.*

The *truth function* is a function that can take any statement as an input. If a statement is true, the value of the function is one.

$$T(statement) = 1$$

If a statement is false, the value of the function is zero.

$$T(statement) = 0$$

Example 11.1

- *Let s be the following statement: $s =$* `A knight and a liar can give the same answer to the same question.` *Then $T(s) = 1$. See problem 10.4.*

- *Let s be the following statement: $s =$* `An inhabitant of the Island of Knights and Liars called himself a liar.` *Then $T(s) = 0$. See problem 10.7.*

Problem 11.5 *Find the values of the truth function for the statements below.*

- $s =$ *A mobster is a big lobster.* $T(s) =$ _____

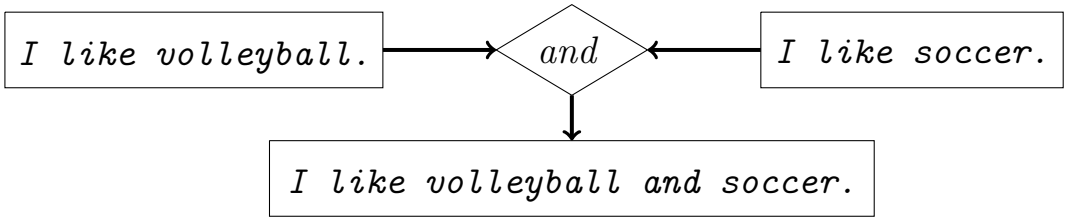
- $s =$ *To lie with consistency, one needs truly good memory.* $T(s) =$ _____

- $s =$ *The word polygon means a* $3D$ *solid in Ancient Greek.* $T(s) =$ _____

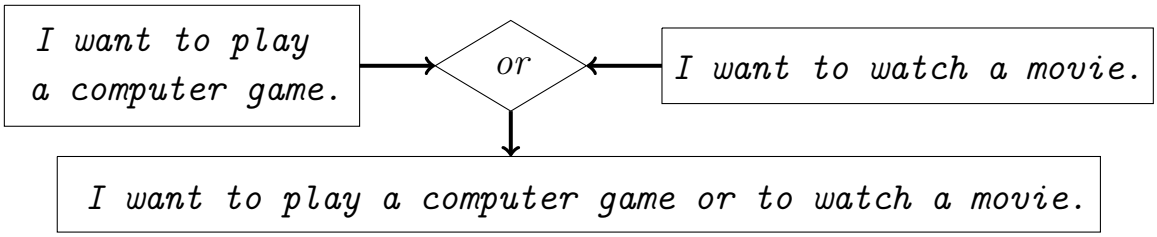
Simple and composite statements

A statement is called *simple* when it cannot be broken down into other statements. For example, the statement *I am eight.* is simple. A statement is called *composite* when it is formed by other statements connected by *logic operations* such as *and*, *or*, *if ... then*, etc. Here are a few examples of composite statements.

I like volleyball and soccer. This statement is composed of two simple statements, *I like volleyball.* and *I like soccer.* connected by the logic operation *and*.



The statement *I want to play a computer game or to watch a movie.* is composed of two simple statements, *I want to play a computer game.* and *I want to watch a movie.* connected by the logic operation *or*.



The statement *If the weather is good, then we will go to the beach today.* is composed of two simple statements, *The weather is good.* and *We will go to the beach today.* connected by the logic operation *if ... then*. The operation is called *implication*. We are not going to draw a chart for this one.

Problem 11.6 *Decide whether the statements below are simple or composite. Circle correct answers. Explain your choices. The problem continues to the next page.*

- *Children went to see a movie.*

Simple *Composite*

- *Alice and Bob went to see a movie.*

Simple *Composite*

- *Cindy stays at home.*

Simple *Composite*

- *Cindy waters the garden.*

Simple *Composite*

- *If Cindy stays at home, she waters the garden.*

Simple *Composite*

- *Either Cindy or David stays at home.*

Simple *Composite*

Problem 11.7 *Write down the simple statements making up the composite statements below. The problem continues to the next page.*

- *Emily will be back in a day or two.*

First simple statememnt: _____

_____

Second simple statememnt: _____

_____

- *Everyone must know how to read and write.*

First simple statememnt: _____

_____

Second simple statememnt: _____

_____

- *If you leave now, you will miss the game.*

First simple statememnt: _____

_____

Second simple statememnt: _____

_____

The truth function and the *and* operation.

Suppose the logic operation *and* is used to make a composite statement out of two other statements. Problems 11.8 and 11.9 help us understand how the truthfulness of the composite statement depends on the truthfulness of its parts.

Recall that a statement s being true is equivalent to $T(s) = 1$. A statement s being false is equivalent to $T(s) = 0$.

Problem 11.8 *Find the values of the truth function for the statements below.*

- $s =$ `Sky is blue.` $T(s) =$ _____
- $s =$ `Sky is green.` $T(s) =$ _____
- $s =$ `Grass is pink.` $T(s) =$ _____
- $s =$ `Grass is green.` $T(s) =$ _____

Statements in the next problem are composed of the statements from problem 11.8 with the help of the operation *and*.

Problem 11.9 *Find the values of the truth function for the statements below. The problem continues to the next page.*

- $s =$ `Sky is green and grass is pink.` $T(s) =$ _____
- $s =$ `Sky is green and grass is green.` $T(s) =$ _____
- $s =$ `Sky is blue and grass is pink.` $T(s) =$ _____
- $s =$ `Sky is blue and grass is green.` $T(s) =$ _____

Out of the four composite statements considered above, the truth function takes the value 1 just once. This happens when both statements making up the composite statement are true. We see an important law of logic at work.

Let a composite statement be made of two other statements with the help of the logic operation *and*. Then the composite statement is true only if each of the statements that make it up is true. In all other cases, the composite statement is false.

$T(s_1)$	$T(s_2)$	$T(s_1\ and\ s_2)$
0	0	0
1	0	0
0	1	0
1	1	1

(11.1)

Problem 11.10 *A knight from the Island of Knights and Liars makes a statement s_1. A liar from the island makes a statement s_2. A tourist combines both statements into a new statement:*

$$s_1\ and\ s_2$$

Find the value of the truth function on the latter statement.

Answer: $T(s_1\ and\ s_2) =$ _____

Problem 11.11 *Looking for the City of Knights on the Island of Knights and Liars, a tourist came to a place where the road forked. She knew that one of the roads at the fork was going to the City of Knights while the other was going to the City of Liars. There was no sign at the fork to point the tourist in the right direction. Fortunately, there was an islander passing by. He seemed to be in a hurry, so the tourist had time for only one question. What question should the tourist ask to figure out her way?*

Answer: _____

_____

_____

11.2 Homework

Finish solving all the problems from class. Discuss problem 11.1 with your parents. Explain to them that there are two possibly correct answers, depending on the model you choose.

Problem 11.12 *Circle the sentences below that can serve as inputs of the truth function. Do not circle other sentences. Explain your choices.*

- *I am tired.*
- *Fleur and George are tired.*
- *Are you tired?*
- *Don't talk in class unless your teacher asks you to!*

Problem 11.13 *Decide whether the statements below are simple or composite. Circle correct answers. Explain your choices.*

- *This statement is clear and simple.*

Simple *Composite*

- *This is a composite statement.*

Simple *Composite*

- *Every statement is either simple or composite.*

Simple *Composite*

Problem 11.14 *A knight from the Island of Knights and Liars makes a statement s_1. Another knight makes a statement s_2. A tourist puts both statements together into a new statement:*

$$s_1 \text{ and } s_2$$

Find the value of the truth function on the latter statement.

Answer: $T(s_1 \text{ and } s_2) =$ _____

Problem 11.15 *Write down the simple statements making up the composite statements below.*

- *Jake's favorite topics this far are cyphers and logic.*

First simple statememnt: _____

_____

Second simple statememnt: _____

_____

- *Kate will major either in math or in computer science.*

First simple statememnt: _____

_____

Second simple statememnt: _____

_____

Question 11.1 *What is an anagram?*

The word *isle* means a *small island*.

Problem 11.16 *Find an anagram of the word* <u>*isle*</u>*.*

Answer: *isle* $\longleftrightarrow$ _____

Problem 11.17 *1000 members of the parliament of the Island of Knights and Liars gather for a session. At the session, each of them tells the others, "You are all liars!" How many liars are there among them?*

Answer: _____