ORMC Olympiad Group
Spring: Week 1
Geometry: Similarity and Triangles

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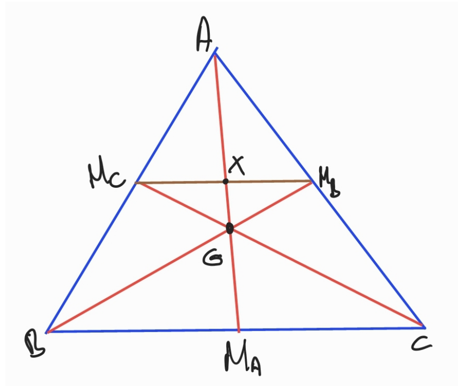
Theorem 1 (**Basic Similarity and Congruence Rules**). *There are a number of ways to find similariry.*

1. *AA*
2. *SAS*
3. *SSS*

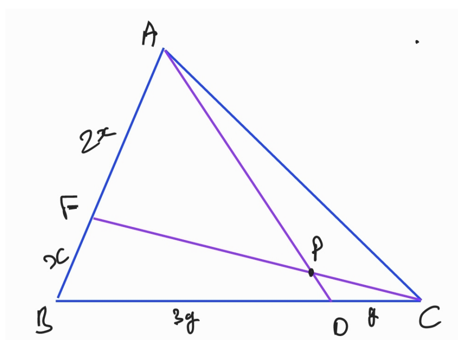
Problems

1. **(Prasolov 1.6 - modified)** A point P is taken on side AD of parallelogram $ABCD$ so that $AP : AD = 1 : 7$; let Q be the intersection point of AC and BP. Find $AC : AQ$.

2. Let M_A, M_B, M_C be the midpoints of the sides BC, CA and AB respectively in triangle ABC. AM_A and M_BM_C intersects at X. Find $AX : XG : GM_A$. Here G is centroid of ABC. In particular, what is AG/GM_A?



3. Points F and D are chosen on the sides AB and BC of triangle ABC. AD and CF intersect at the point P. It is given that $AB/BF = BD/DC = 3$.
   (a) what is AP/PD?
   (b) What is CP/PF?



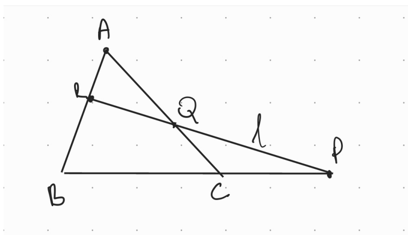
4. The point D is chosen on the side AC of triangle ABC. If $AB = 6$, $AD = 4$, $\angle BAC = 40$, $\angle ACB = 60$, $\angle DBC = 20$, find DC.
5. **(Prasolov 0.5)** Square $PQRS$ is inscribed into $\triangle ABC$ so that vertices P and Q lie on sides AB and AC and vertices R and S lie on BC. Express the length of the square's side in terms of a and h_a.
6. ABC is an equilateral triangle with side length 15. Points D, E, F are chosen on the sides BC, CA and AB respectively so that $BD = CE = AF = 7$. When we draw AD, BE, CF, we create a smaller equilateral triangle in the middle of ABC, say that $\triangle XYZ$. Side lenght

of $\triangle XYZ$ can be represented as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

7. **(HMMT 2005 Guts)** Five people of different heights are standing in line from shortest to tallest. As it happens, the tops of their heads are all collinear; also, for any two successive people, the horizontal distance between them equals the height of the shorter person. If the shortest person is 3 feet tall and the tallest person is 7 feet tall, how tall is the middle person, in feet?

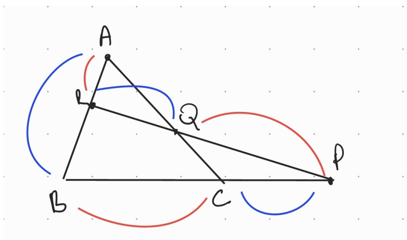
8. Let ABC be triangle with $AB = 6, AC = 7, BC = 8$, and P is a point on BC with $BP = 3$. Let Q and R be on sides AC and AB so that $PQ \parallel AB$ and $PR \parallel AC$. The are of the parallelogram $AQPR$ can be written as $\frac{p\sqrt{q}}{r}$ where p and r are relatively prime and q is square-free integer. Find $p + q + r$.

9. **Menelaus' Theorem** ABC is a triangle. A line l cuts the segments AB and AC at R and Q, and cuts the extension of BC at P.



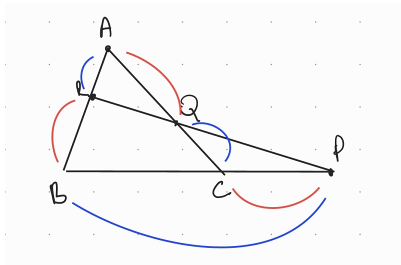
Then

$$\frac{AR}{AB} \cdot \frac{BC}{CP} \cdot \frac{PQ}{QR} = 1$$



and similarly

$$\frac{PC}{PB} \cdot \frac{BR}{RA} \cdot \frac{AQ}{QC} = 1$$



10. **Ceva's Theorem** ABC is a triangle and P is an interior point. The cevians AP, BP, CP cuts the sides BC, CA, AB at points at A_1, B_1, C_1 resppectively. Then
$$\frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} \cdot \frac{AC_1}{C_1B} = 1$$

11. Let ABC be a triangle with $BC = 70$ and points M and N are chosen on the sides AB and Ac so that $MN \parallel BC$. Segments CM and BN intersect at the point K. A line which passes through K and parallel to BC intersects with the sides AB and AC at X and Y. Find MN if $XY = 42$.

12. **(Prasolov 1.13)** In $\triangle ABC$ bisectors AA_1 and BB_1 are drawn. Prove that the distance from any point M of A_1B_1 to line AB is equal to the sum of distances from M to AC and BC.

13. **(TJNMO-FR 2017-modified)** Point E is chosen in a parallelogram $ABCD$ so that $\angle AEB + \angle DEC = 180°$. Prove that $\angle DAE = \angle DCE$