Logic Gates Intro Lessons 1-5 Compiled

1 Knights and liars

Problem 1.1 *One week, Mrs. Jones sold eggs at the market every day from Monday through Friday. She sold 73 eggs on Thursday and 118 on Friday. Doing her Saturday bookkeeping, Mrs. Jones noticed that the number of the eggs sold every day of that week starting with Wednesday was equal to the sum of the numbers of the eggs sold in the previous two days. How many eggs did Mrs. Jones sell on Monday? Filling out the table below helps to answer the question.*

Day	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
# of eggs				73	118

Answer: Mrs. Jones sold ________ eggs on Monday.

There are two kinds of people living on the Island of Knights and Liars. A *knight* always tells the truth. A *liar* always lies.

Problem 1.2 *An islander tells you that the top and bottom projections of a solid are mirror images of each other. Is this a knight or a liar? Circle the correct answer. Explain your choice.*

Knight Liar

Problem 1.3 *You ask a knight the same question twice and get two different answers. What could be the question?*

Answer: ____

Problem 1.4 *You ask a knight and a liar the same question and get the same answer. What could be the question?*

Answer: ____

Problem 1.5 *Can you ask an islander exactly one question to find out if this is a knight or a liar? What could be the question?*

Answer: ____

Problem 1.6 *An islander claims that* $1 + 1 = 10$. *Is this a knight or a liar? Circle the correct answer. Explain your choice.*

Knight Liar Could be either

There are *tourists* visiting the Island of Knights and Liars. A tourist sometimes tells the truth and some other times lies.

Problem 1.7 *Alice says, "I am a liar." What kind is she? Circle the correct answer. Explain your choice.*

Knight Liar Tourist

Problem 1.8 *Bob says, "I am a knight." Is it possible to figure out what kind he is? Circle the correct answer. Explain your choice.*

Yes No

Problem 1.9 *Charlie, Donna, and Emily are islanders, not tourists. A tourist asked Charlie if he was a knight. Charlie mumbled something chewing his gum, so the tourist did not understand his answer. The tourist asked Donna what Charlie said. Donna answered that Charlie claimed to be a liar. To that Emily responded, "Don't believe Donna, she is a liar herself!" What kinds are Charlie, Donna, and Emily?*

Answer: Charlie ________________ ,

Donna is a _____________ ,

Emily is a _____________ .

Problem 1.10 *Fleur and George live on the Island of Knights and Liars. George says, "We are both liars." Who is Fleur and who is George? Note: since they live on the island, they are not tourists.*

Answer: Fleur is a _________________ ,

George is a _________________ .

Problem 1.11 *Henry and Irene live on the Island of Knights and Liars. Henry says, "We are the same kind." Irene says, "We are different kinds." Who is Henry and who is Irene?*

Answer: Henry is a _________________ ,

Irene is a _________________ .

Problem 1.12 *The following way of punishing criminals was used for a while on the Island of Knights and Liars. A convicted lawbreaker would be brought to two neighboring identically looking doors leading to two jail cells. One of the cells was empty. There was a hungry tiger in the other cell. There were two guards standing next to the doors. One of the guards was a knight, the other was a liar, but there was no way to tell what kind they were based on their looks only. The criminal was allowed to ask one guard one question and then to choose the door based on the answer. What question would you ask? How would you choose the door?*

Question: ______________________________

____

____

Choosing: ______________________________

Finish solving all the problems from class. Teach your parents about the Island of Knights and Liars.

Problem 1.13 *A tourist meets two islanders, Jake and Kate, and asks them if they are knights or liars. Jake says, "If you ask Kate whether she is a knight, she would say no." Kate says, "If you ask Jake whether he is a knight, he would say yes." Who are they?*

Answer: Jake is a ______________ ,

Kate is a ______________ .

Problem 1.14 *A tourist meets two islanders, Luke and Mary, and asks them if they are knights or liars. Luke says, "If you ask Mary* *whether I am a knight, she would say no." Mary says, "If you ask Luke whether I am a knight, he would say yes." Who are they?*

Answer: Luke is a ________________ ,

Mary is a ________________ .

Problem 1.15 *The age of Nick's great grandfather is the smallest three-digit number written with three different digits. How old is Nick's great grandpa?*

Answer: ____________

Problem 1.16 *Olivia noticed that the sum of her age and the ages of her mom and dad equals to 70. The girl is wondering when the sum of the ages will be equal to 100. Can you help her?*

Answer: ______________________________________

Problem 1.17 *One apple costs more than two bananas. What is more expensive, two apples or three bananas?*

Answer: ____

2 The truth function and the *and* operation

Problem 2.1 *It is raining at midnight on Tuesday. Do you think we can expect sunny weather in 48 hours? Circle the correct answer. Explain your choice.*

Yes *No* *Hard to tell*

A *statement* is a sentence which is either *True* or *False*. For example, `I am a knight.` is a statement. `Are you a liar?` is not.

Problem 2.2 *Decide which sentences below are statements and which are not. Circle correct answers. Explain your choices. The problem continues to the next page.*

- `A knight always tells the truth.`

  Statement *Not statement*

- `A liar always tells the truth.`

  Statement *Not statement*

- `Does a tourist always tell the truth?`

  Statement *Not statement*

- `Smoking is forbidden on UCLA campus.`

  Statement *Not statement*

- `Please refrain from smoking on board of the aircraft.`

Statement *Not statement*

- $11 + 1 = 100$

Statement *Not statement*

- `This statement is a lie.`

Statement *Not statement*

Problem 2.3 *Use the lines below to write a sentence that is a statement.*

____

____

Problem 2.4 *Use the lines below to write a sentence that is not a statement.*

____

____

The *truth function* is a function that can take any statement as an input. If a statement is true, the value of the function is one.

$$T(statement) = 1$$

If a statement is false, the value of the function is zero.

$$T(statement) = 0$$

Example 2.1

• *Let s be the following statement:* $s =$ `A knight and a liar can give the same answer to the same question.` *Then* $T(s) = 1$. *See problem 1.4.*

• *Let s be the following statement:* $s =$ `An inhabitant of the Island of Knights and Liars called himself a liar.` *Then* $T(s) = 0$. *See problem 1.7.*

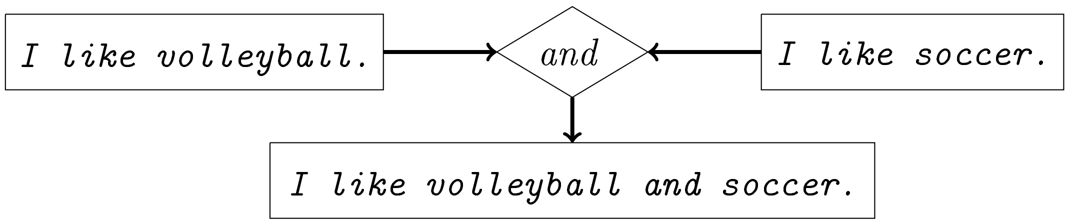
Problem 2.5 *Find the values of the truth function for the statements below.*

- $s =$ `A mobster is a big lobster.` $T(s) =$ ________
- $s =$ `To lie with consistency, one needs truly good memory.` $T(s) =$ ________
- $s =$ `The word` *polygon* `means a` $3D$ `solid in Ancient Greek.` $T(s) =$ ________

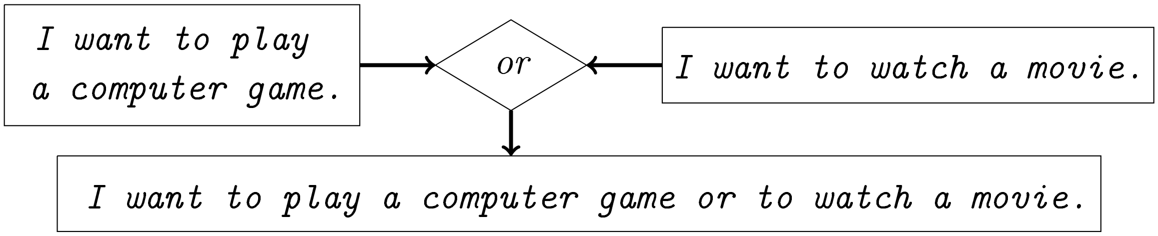
Simple and composite statements

A statement is called *simple* when it cannot be broken down into other statements. For example, the statement `I am eight.` is simple. A statement is called *composite* when it is formed by other statements connected by *logic operations* such as *and*, *or*, *if ... then*, etc. Here are a few examples of composite statements.

I like volleyball and soccer. This statement is composed of two simple statements, *I like volleyball.* and *I like soccer.* connected by the logic operation *and*.



The statement *I want to play a computer game or to watch a movie.* is composed of two simple statements, *I want to play a computer game.* and *I want to watch a movie.* connected by the logic operation *or*.



The statement *If the weather is good, then we will go to the beach today.* is composed of two simple statements, *The weather is good.* and *We will go to the beach today.* connected by the logic operation *if ... then*. The operation is called *implication*. We are not going to draw a chart for this one.

Problem 2.6 *Decide whether the statements below are simple or composite. Circle correct answers. Explain your choices. The problem continues to the next page.*

- *Children went to see a movie.*

  $Simple$ $Composite$

- *Alice and Bob went to see a movie.*

  $Simple$ $Composite$

- *Cindy stays at home.*

  $Simple$ $Composite$

- *Cindy waters the garden.*

  $Simple$ $Composite$

- *If Cindy stays at home, she waters the garden.*

  $Simple$ $Composite$

- *Either Cindy or David stays at home.*

  $Simple$ $Composite$

Problem 2.7 *Write down the simple statements making up the composite statements below.*

- `Emily will be back in a day or two.`

First simple statememnt: ______________________________

____

Second simple statememnt: ______________________________

____

- `Everyone must know how to read and write.`

First simple statememnt: ______________________________

____

Second simple statememnt: ______________________________

____

- `If you leave now, you will miss the game.`

First simple statememnt: ______________________________

____

Second simple statememnt: ______________________________

____

The truth function and the *and* operation.

Suppose the logic operation *and* is used to make a composite statement out of two other statements. Problems 2.8 and 2.9 help us understand how the truthfulness of the composite statement depends on the truthfulness of its parts.

Recall that a statement s being true is equivalent to $T(s) = 1$. A statement s being false is equivalent to $T(s) = 0$.

Problem 2.8 *Find the values of the truth function for the statements below.*

- $s =$ `Sky is blue.` $T(s) =$ ________
- $s =$ `Sky is green.` $T(s) =$ ________
- $s =$ `Grass is pink.` $T(s) =$ ________
- $s =$ `Grass is green.` $T(s) =$ ________

Statements in the next problem are composed of the statements from problem 2.8 with the help of the operation *and*.

Problem 2.9 *Find the values of the truth function for the statements below. The problem continues to the next page.*

- $s =$ `Sky is green and grass is pink.` $T(s) =$ ________
- $s =$ `Sky is green and grass is green.` $T(s) =$ ________

- $s =$ `Sky is blue and grass is pink.` $T(s) =$ ________

- $s =$ `Sky is blue and grass is green.` $T(s) =$ ________

Out of the four composite statements considered above, the truth function takes the value 1 just once. This happens when both statements making up the composite statement are true. We see an important law of logic at work.

Let a composite statement be made of two other statements with the help of the logic operation *and*. Then the composite statement is true only if each of the statements that make it up is true. In all other cases, the composite statement is false.

$T(s_1)$	$T(s_2)$	$T(s_1 \textit{ and } s_2)$
0	0	0
1	0	0
0	1	0
1	1	1

(2.1)

Problem 2.10 *A knight from the Island of Knights and Liars makes a statement s_1. A liar from the island makes a statement s_2. A tourist combines both statements into a new statement:*

$$s_1 \textit{ and } s_2$$

Find the value of the truth function on the latter statement.

Answer: $T(s_1 \textit{ and } s_2) =$ ________

Problem 2.11 *Looking for the City of Knights on the Island of Knights and Liars, a tourist came to a place where the road forked. She knew that one of the roads at the fork was going to the City of Knights while the other was going to the City of Liars. There was no sign at the fork to point the tourist in the right direction. Fortunately, there was an islander passing by. He seemed to be in a hurry, so the tourist had time for only one question. What question should the tourist ask to figure out her way?*

Answer: ____

____

____

Problem 2.12 *Circle the sentences below that can serve as inputs of the truth function. Do not circle other sentences. Explain your choices.*

- *I am tired.*
- *Fleur and George are tired.*
- *Are you tired?*
- *Don't talk in class unless your teacher asks you to!*

Problem 2.13 *Decide whether the statements below are simple or composite. Circle correct answers.*

- `This statement is clear and simple.`

Simple   *Composite*

- *This is a composite statement.*

Simple *Composite*

- *Every statement is either simple or composite.*

Simple *Composite*

Problem 2.14 *A knight from the Island of Knights and Liars makes a statement s_1. Another knight makes a statement s_2. A tourist puts both statements together into a new statement:*

$$s_1 \textit{ and } s_2$$

Find the value of the truth function on the latter statement.

Answer: $T(s_1 \textit{ and } s_2) =$ ________

Problem 2.15 *Write down the simple statements making up the composite statements below.*

- *Jake's favorite topics this far are cyphers and logic.*

First simple statememnt: ______________________________________

____

Second simple statememnt: ____________________________________

____

- `Kate will major either in math or in computer science.`

First simple statememnt: ______________________________

____

Second simple statememnt: ______________________________

____

Problem 2.16 *1000 members of the parliament of the Island of Knights and Liars gather for a session. At the session, each of them tells the others, "You are all liars!" How many liars are there among them?*

Answer: ____________

3 The *or* and negation operations

Once upon a time in a land far, far away there lived a very beautiful princess, the only daughter of a very evil king. Some day, a handsome and very smart prince from a neighboring kingdom came to pay them a visit. The princess and prince fell in love with each other and asked the king for a permission to marry. The evil king didn't want his daughter to leave. Instead of blessing the marriage, he ordered to put the prince in jail and to prepare for his execution.

The princess begged the king not to kill the prince and finally he agreed. He told the prisoner, "At her Highness's request, I will give you a chance. Tomorrow you will be brought to my court. You will have to pull a lot. I will put two pieces of paper in the box. One will read LIFE, the other will read DEATH. Whichever you pull out, it will be your destiny."

The king was a very evil man. He ordered his minister of justice to write DEATH on both notes. The princess overheard the king's order and found a way to warn the prince.

Problem 3.1 *What should the prince do to survive? Hint: kings do not like public embarrassment.*

The truth function and the *or* operation.

Suppose the logic operation *or* is used to make a composite statement out of two other statements. Let us reuse the statements from problem 2.8 to figure out how the truthfulness of the composite statement depends on the truthfulness of the parts. Recall that a statement s being true is equivalent to $T(s) = 1$. A statement s being false is equivalent to $T(s) = 0$.

- $s =$ `Sky is blue.` $T(s) =$ ___1___
- $s =$ `Sky is green.` $T(s) =$ ___0___
- $s =$ `Grass is pink.` $T(s) =$ ___0___
- $s =$ `Grass is green.` $T(s) =$ ___1___

Statements in the next problem are composed of the statements above with the help of the operation *or*.

Problem 3.2 *Find the values of the truth function for the statements below.*

- $s =$ `Sky is green or grass is pink.` $T(s) =$ ________
- $s =$ `Sky is green or grass is green.` $T(s) =$ ________
- $s =$ `Sky is blue or grass is pink.` $T(s) =$ ________
- $s =$ `Sky is blue or grass is green.` $T(s) =$ ________

Out of the four composite statements considered above, the truth function takes the value 0 just once. This happens when both statements making up the composite statement are false. We see an important law of logic at work:

Let a composite statement be made of two other statements with the help of the logic operation *or*. Then the composite statement is false only if each of the statements that make it up is false. In all other cases, the composite statement is true.

$T(s_1)$	$T(s_2)$	$T(s_1 \text{ or } s_2)$
0	0	0
1	0	1
0	1	1
1	1	1

(3.2)

Problem 3.3 *A knight from the Island of Knights and Liars makes a statement* s_1*. A liar from the island makes a statement* s_2*. A tourist puts the statements together into a new statement:* s_1 *or* s_2*. Find the value of the truth function on the latter statement.*

Answer: $T(s_1 \text{ or } s_2) =$ ________

Problem 3.4 *A liar from the Island of Knights and Liars makes a statement* s_1*. Another liar makes a statement* s_2*. A tourist combines the statements together into a new statement:* s_1 *or* s_2*. Find the value of the truth function on the latter statement.*

Answer: $T(s_1 \text{ or } s_2) =$ ________

Negation

Negation is a function that takes any statement s as an input and produces the opposite statement $\bar{s}$ as the output. For example, let

$$s = \texttt{A knight always tells the truth.}$$

Then

$$\bar{s} = \texttt{A knight does not always tell the truth.}$$

Less formally, to negate a statement means to say, "No, no, no, it's the exact opposite of what you just said!" To practice, let us negate a few statements we have seen before.

Problem 3.5

- $s =$ `A mobster is a big lobster.`

  $\bar{s} =$ ______________________________

- $s =$ `The word` *polygon* `means a 3D solid in Ancient Greek.`

  $\bar{s} =$ ______________________________

  ____

- $s =$ `Children went to see a movie.`

  $\bar{s} =$ ______________________________

Let $\bar{s}$ be the negation of a statement s. Since $\bar{s}$ claims the exact opposite of what s claims, the truth function must take different values on s and $\bar{s}$. If s is true, then $\bar{s}$ must be false and the other way around. For example, consider

$s =$ `A mobster is a big lobster.` and

$\bar{s} =$ `A mobster is not a big lobster.`

In this case, $T(s) = 0$ and $T(\bar{s}) = 1$. Let us summarize with a negation truth table.

$T(s)$	$T(\bar{s})$
0	1
1	0

(3.3)

Problem 3.6 *Find $T(\bar{s})$ for the statements s given below.*

- $s =$ `Sky is blue.` $T(\bar{s}) =$ ________
- $s =$ `Sky is green.` $T(\bar{s}) =$ ________
- $s =$ `Grass is pink.` $T(\bar{s}) =$ ________
- $s =$ `Grass is green.` $T(\bar{s}) =$ ________
- $s =$ `Sky is blue and grass is pink.` $T(\bar{s}) =$ ________
- $s =$ `Sky is blue or grass is pink.` $T(\bar{s}) =$ ________

Problem 3.7 *A knight from the Island of Knights and Liars makes a statement s_1. A liar from the island makes a statement s_2. A tourist combines both statements into a new statement*

$$s = s_1 \text{ and } s_2$$

and then negates it. Is the negated statement true or false? Circle the correct answer. Explain your choice.

True *False*

Problem 3.8 *Alice, Bob, and Charlie live on the Island of Knights and Liars.*

- *Alice says that all three of them are liars.*
- *Bob agrees with her.*
- *Charlie says that only two of them are liars.*

What kind are they?

Answer: Alice is a ____________ ,

Bob is a ____________ ,

Charlie is a ____________ .

Problem 3.9 *One year, there were exactly four Tuesdays and Saturdays in July. What day was July 31st?*

Answer: ________________

Tell your parents the story about a handsome prince, a beautiful princess, and an evil king from earlier. Then explain how logic helped the prince to survive.

Problem 3.10 *The problem continues to the next page.*

- *Write down the statements s_1 and s_2 making up the composite statement s below.*

$s =$ `Jupiter or Earth is the largest planet of the Solar system.`

$s_1 =$ ____

____

$s_2 =$ ____

____

- *Write down the formula for s in terms of s_1, s_2, and a proper logic operation.*

$$s = \underline{\hspace{4cm}}$$

- *Find the values the truth function takes on the statements s_1 and s_2.*

$$T(s_1) = \underline{\hspace{2cm}} \qquad\qquad T(s_2) = \underline{\hspace{2cm}}$$

- *Find the values the truth function takes on the statements s and $\bar{s}$.*

$$T(s) = \underline{\hspace{2cm}} \qquad\qquad T(\bar{s}) = \underline{\hspace{2cm}}$$

Problem 3.11 *A liar from the Island of Knights and Liars makes a statement s_1. Another liar makes a statement s_2. A tourist combines both statements into a new statement $s = s_1$ or s_2 and then negates it. Is the negated statement true or false? Circle the correct answer. Explain your choice.*

True   *False*

Let us continue the story about a handsome prince, a beautiful princess, an evil king, and the usefulness of logic.

The father of the prince, the king of the neighboring country, had his peculiarities, too. The old man invented the following way of punishing criminals. Lawbreakers were given a choice between two doors. Behind each door, there was either a hungry tiger or a treasure of gold, but not nothing or both. The king would also post some warnings on the doors and then let the criminals choose.

Problem 3.12 *One day, there was a criminal facing the doors with the following signs.*

Door 1: at least one of these rooms has gold.

Door 2: a tiger is in the other room.

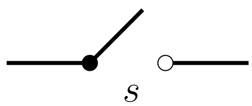
"Are the signs true?" asked the prisoner. "They are either both true or both false," replied the king. Then he smiled warmly and added, "Make your choice, buddy!"

Which door should the prisoner open? Why?

Answer: ________________

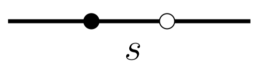
4 Logic gates

The picture on the left-hand side below is a scheme of an electric switch. The black circle is an entry point. The white circle is an exit point. The switch is off, so there is no electric current from the entry point to the exit one. The switch represents a false statement. The value 0 the truth function takes on a false statement is equivalent to the absence of current through the switch.



s is false, $T(s) = 0$

The next picture is a scheme showing a switch that is on. Since the switch is on, there is an electric current from the entry point to the exit point. The switch represents a true statement. The value 1 the truth function takes on a true statement is equivalent to the presence of current through the switch.

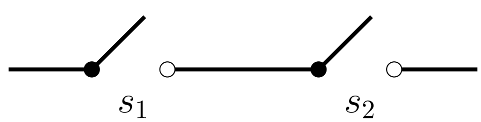


s is true, $T(s) = 1$

The *and* gate

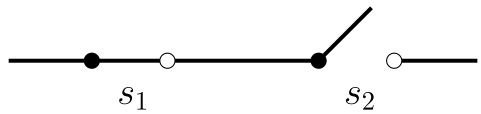
Let us show that two consecutive switches on a wire form a *logic gate* corresponding to the operation *and*.

Case 1: two false statements correspond to two switches that are off.



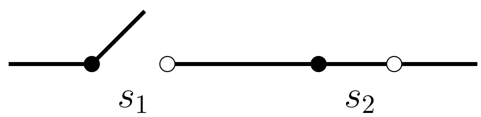
There is no current through the gate. If $T(s_1) = 0$ and $T(s_2) = 0$, then $T(s_1 \textit{ and } s_2) = 0$.

Case 2: if the statement s_1 is true and the statement s_2 is false, we get the following scheme:



There is no current through the gate. If $T(s_1) = 1$ and $T(s_2) = 0$, then $T(s_1 \text{ } and \text{ } s_2) = 0$. We consider the third case in the next problem.

Problem 4.1 *Case 3: take a look at the scheme of the* <u>*and*</u> *gate below. Then answer the questions that follow the picture. The problem continues to the next page.*



- *Is the statement s_1 true or false? Circle the correct answer. Explain your choice.*

True *False*

- *Is the statement s_2 true or false? Circle the correct answer. Explain your choice.*

True *False*

- *What are the values the truth function takes on the statement s_1 and the statement s_2?*

$T(s_1) =$ ________ $T(s_2) =$ ________

- *Is there a current through the gate? Circle the correct answer. Explain your choice.*

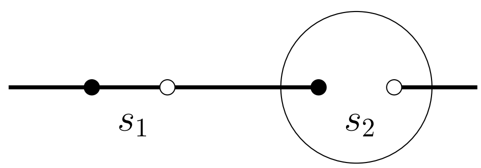
$$\textit{Yes} \qquad\qquad\qquad \textit{No}$$

- *What value does the truth function take on the statement $\underline{s_1 \text{ and } s_2}$?*

$$T(s_1 \textit{ and } s_2) = \underline{\qquad\qquad}$$

We consider the fourth case for the *and* gate in the next problem.

Problem 4.2 *Case 4: Given $T(s_1$ and $s_2) = 1$, draw the switch s_2 in the bubble on the picture below.*



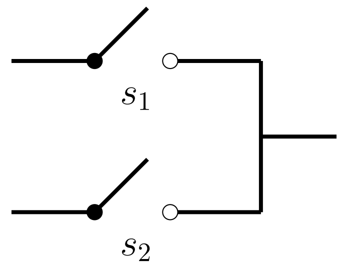
- *What are the values the truth function takes on the statement s_1 and the statement s_2?*

$$T(s_1) = \underline{\qquad\qquad} \qquad\qquad T(s_2) = \underline{\qquad\qquad}$$

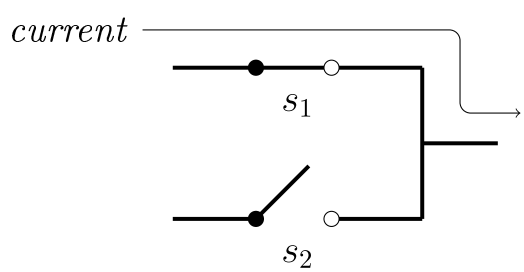
Comparing the four cases above to the truth table (11.1) for the operation *and*, we see that the gate is equivalent to the operation.

The *or* gate

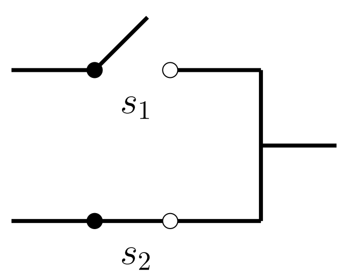
Case 1: the following is a scheme of an *or* gate having two false statements as inputs. This time, the switches are parallel. If $T(s_1) = 0$ and $T(s_2) = 0$, then there is no current through the gate. Thus, $T(s_1 \textit{ or } s_2) = 0$.



Case 2: the below is the scheme of an *or* gate with $T(s_1) = 1$ and $T(s_2) = 0$. The switch s_1 is on, the switch s_2 is off. Since the switches are parallel, the current can pass the gate. Thus, $T(s_1 \textit{ or } s_2) = 1$.



Problem 4.3 *Case 3: take a look at the scheme below. Then answer questions on the next page.*



• *Is there a current through the gate? Circle the correct answer. Explain your choice.*

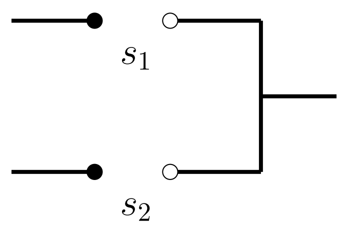
Yes *No*

- *If $T(s_1) = 0$ and $T(s_2) = 1$, what value does the truth function take on the statement $\underline{s_1 \text{ or } s_2}$?*

$$T(s_1 \text{ or } s_2) = \underline{\hspace{2cm}}$$

Problem 4.4 *Case 4: $T(s_1) = 1$ and $T(s_2) = 1$.*

- *Draw the switches s_1 and s_2 on the scheme below.*



- *Is there a current through the gate? Circle the correct answer.*

Yes *No*

- *If $T(s_1) = 1$ and $T(s_2) = 1$, what value does the truth function take on the statement $\underline{s_1 \text{ or } s_2}$?*

$$T(s_1 \text{ or } s_2) = \underline{\hspace{2cm}}$$

Comparing the four cases above to the truth table (12.2) for the operation *or*, we see that the gate is equivalent to the operation.

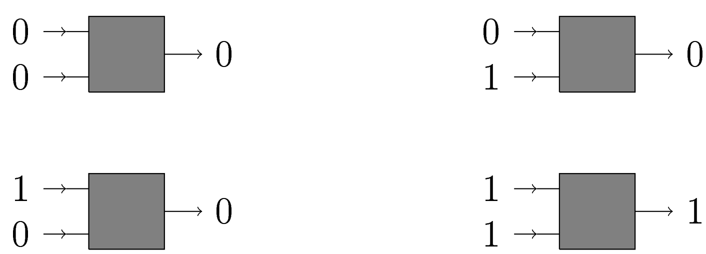
The *not* gate

The *not gate*, also called an *inverter*, is equivalent to the operation of negation. An inverter has one incoming wire corresponding to a statement s and one outgoing wire corresponding to the negated statement $\bar{s}$. If there is current in the incoming wire, there is no current in the outgoing wire and the other way around. Equivalently, if $T(s) = 1$, then $T(\bar{s}) = 0$. If $T(s) = 0$, then $T(\bar{s}) = 1$.

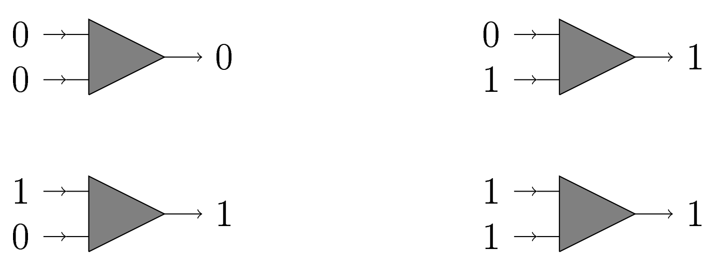
Pictorial representation

We use the following pictures for the inverter.

We use the following pictures for the *and* gate.

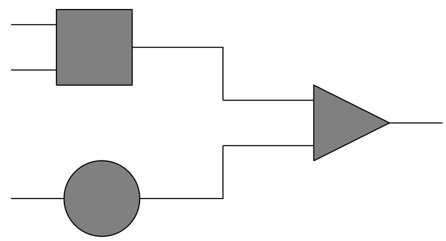


We use the following pictures for the *or* gate.



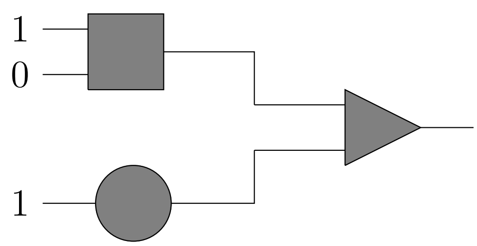
In what follows, we assume that the current always flows left-to-right and do not draw arrows.

A few logic gates can be combined in a *logic circuit*. Here is an example:



Example 4.1 *Let us find the output of the above logic circuit with the inputs given on the picture below.*

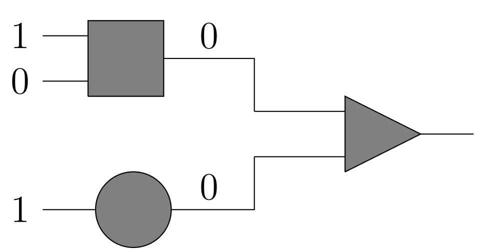
• *At the top of the picture, there is an* and *gate, denoted by a square. The inputs of the gate are two wires, one with current and one without it. There is no current in the output wire. Please see case 2 on page 26.*



The gate has performed the following computation.

a true statement and *a false statement = a false statement*

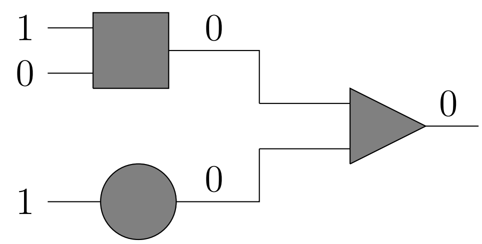
• *The inverter at the bottom of the picture is denoted by a circle. The input of the inverter is a true statement as indicated by a wire with current in it. The output is a false statement, indicated by the absence of current in the output wire.*



The gate has performed the following computation.

a true statement <u>*negated*</u> = *a false statement*

• *The outputs of the* <u>*and*</u> *gate and of the inverter are the inputs of the* <u>*or*</u> *gate on the right-hand side of the picture. There is no current in both incoming wires, so there is no current in the output wire either.* *See case 1 on page 27.*

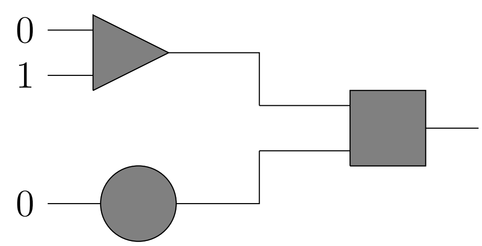


The gate has performed the following computation.

a false statement <u>*or*</u> *a false statement* = *a false statement*

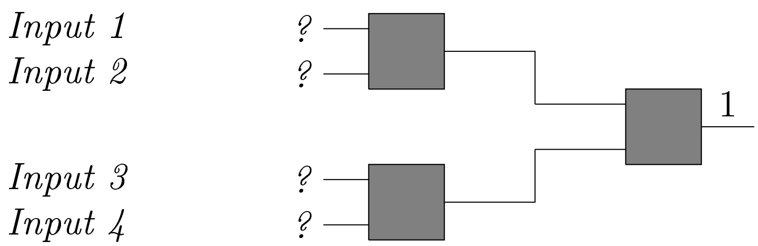
The output of the circuit is 0.

Problem 4.5 *Find the output of the following circuit.*



Answer: ________

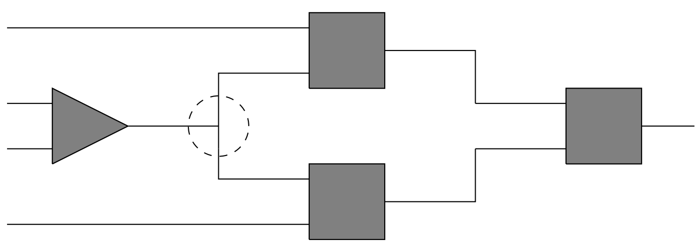
Problem 4.6 *The output of the circuit below is* 1. *What are the inputs?*



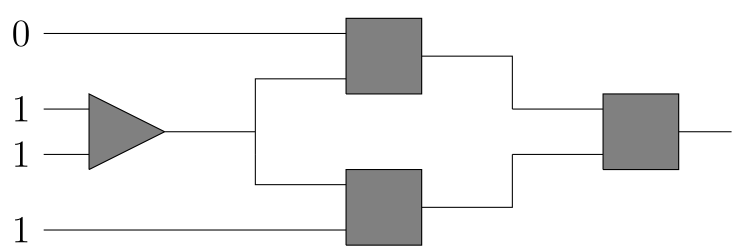
Input 1 = ________ *Input 2* = ________

Input 3 = ________ *Input 4* = ________

The output wire of a logic gate can be split to feed a few more logic gates. See the output of the *or* gate in the dashed bubble on the picture below.

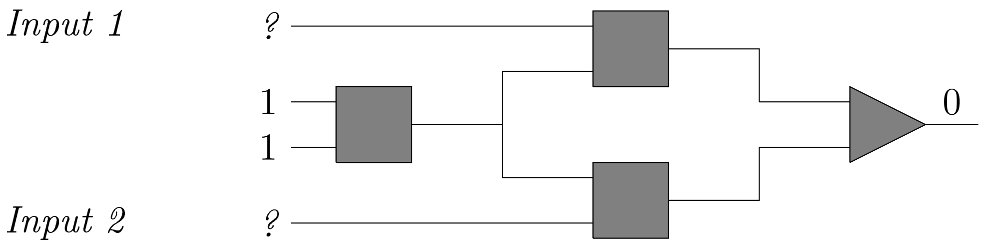


Problem 4.7 *Find the output of the circuit below.*



Answer: ________

Problem 4.8 *The output of the circuit below is* 0*. What are the missing inputs?*

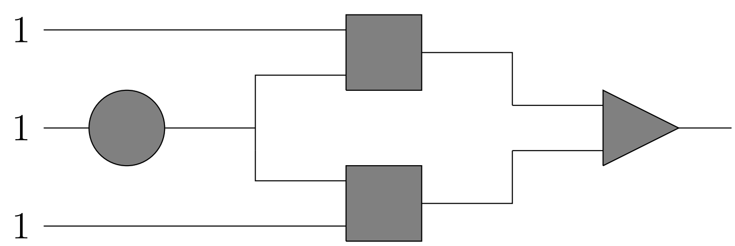


Input 1 = ________ *Input 2* = ________

Problem 4.9 *Can an intersection of a triangle and a quadrilateral be an octagon? If you think it can, draw an example in the space below. If you think it can't, explain why not.*

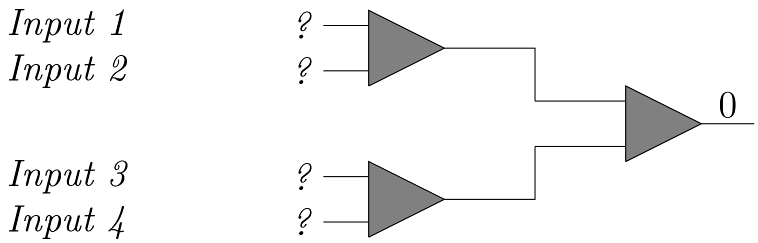
Teach your parents about logic gates. Show them how the *and*, *or*, and *not* gates work.

Problem 4.10 *What is the output of the circuit below?*



Answer: ________

Problem 4.11 *The output of the circuit below is 0. What are the inputs?*



Input 1 = ________ *Input 2* = ________

Input 3 = ________ *Input 4* = ________

We are back in jail with the king from problem 3.12 and with a new prisoner. As in the past, there are two doors. There is either a hungry tiger or a treasure of gold behind each door, but not nothing or both.

Problem 4.12 *The prisoner has to choose one of the two doors in front of him. As before, the king has placed signs on the doors.*

Door 1: either there is a tiger in this room, or there is gold in the other room.

Door 2: there is gold in the other room.

Once again, the king tells the prisoner that the messages are either both true or both false. Which door should the prisoner open? Why?

Answer: ____

5 Double negation

The negation of the statement

$$s = \texttt{Sky is blue.}$$

is the statement

$$\bar{s} = \texttt{Sky is not blue.}$$

The negation of the latter statement is the statement

$$\bar{\bar{s}} = \texttt{Sky is blue.}$$

We see that $\bar{\bar{s}} = s$.

Problem 5.1 *For each statement s below, write down its negation $\bar{s}$ and its double negation $\bar{\bar{s}}$. Then decide if $s = \bar{\bar{s}}$.*

- $s = \texttt{Bob likes football.}$

$\bar{s} =$ ____

____

$\bar{\bar{s}} =$ ____

____

Is $s = \bar{\bar{s}}$? Circle the correct answer.

Yes *No*

- $s = \texttt{Jupiter is the largest planet of the Solar system.}$

$\bar{s} =$ ____

____

$\bar{\bar{s}} =$ ____

____

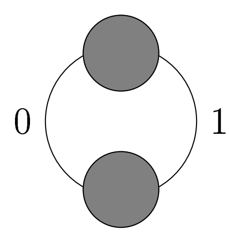
Is $s = \bar{\bar{s}}$? Circle the correct answer.

Yes *No*

We see the following law of logic at work.

A statement negated twice is the same as the original statement.

The law explains the reason why we use a circular symbol for the inverter. The gate works the same both ways!



Problem 5.2 *There are 100 soldiers in a company. Every four hours, three soldiers go on sentry duty. Is it possible to arrange it in such a way that after some period of time every soldier has been on sentry duty with another soldier exactly once? If you think it is possible, please show how. If you think it is not possible, please explain why not.*

Answer: ____________________

Problem 5.3 *For each statement s below, write down its negation $\bar{s}$ and its double negation $\bar{\bar{s}}$. Then decide if $s = \bar{\bar{s}}$.*

- $s =$ `Cindy does not like chocolate.`

$\bar{s} =$ ____

____

$\bar{\bar{s}} =$ ____

____

Is $s = \bar{\bar{s}}$? Circle the correct answer.

Yes *No*

- $s =$ `It's hard to be a king!`

$\bar{s} =$ ____

____

$\bar{\bar{s}} =$ ____

____

Is $s = \bar{\bar{s}}$? Circle the correct answer.

Yes *No*

Problem 5.4 *There are only islanders working at the post office on the Island of Knights and Liars. Each of them works a different amount of time and is paid a different wage.*

One day, a tourist asked all the post office employees two questions.

Question 1: how many people at this office work more than you?

Question 2: how many people at this office get paid better than you?

All the employees gave the same answers to the questions.

Answer 1: at most ten people work more than I do.

Answer 2: at least 20 people are paid better.

How many knights and how many liars work at the office?

Answer: there are ________ knights and ________ liars.