ARCHIMEDEAN SOLIDS

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Definition 1. An archimedean solid is a convex polytope such that each side is a regular polygon and the same polygons meet at every corner in the same order and under the same angles. It is a platonic solid if all the sides are the same regular polygon.

We denote an archimedean solid by a sequence $(p_1, p_2, \ldots, p_n)$ which expresses that at every corner is shared by an p_1-gon, p_2-gon,..., p_n-gon in that order. While every archimedean solid

Problem 1. Give as many examples of archimedean and platonic solids as you can think of.

Problem 2. We now classify all platonic solids. Let such a platonic solid be comprised of regular p-gons such that n sides meet at each vertex.

1. Calculate the sum S of all the angles meeting at a single vertex in terms of n and p.
2. Argue why $S < 360$, $n > 3$, and $p > 3$.
3. Find all possible solutions for n and p.
4. Visualize these platonic solids.

Problem 3. Let P be a platonic solid. We can form its dual solid by placing a vertex in the middle of every side and connecting two vertices if their corresponding sides share a common edge. What are the duals of the five platonic solids?

Problem 4. Let P be a platonic solid. We can obtain a new solid if we "cut off" every vertex. This process is called truncation.

1. Visualize this.
2. Describe exactly how we need to truncate a platonic solid to obtain an archimedean solid.

Problem 5. Describe all the archimedean solids you can obtain from the platonics via truncation.

Problem 6. Let P be a truncated platonic solid. Imagine you slide the plane of truncation around every vertex inwards until the original edges of the platonic vanish. This process is called rectification.

1. Argue that rectification of platonic solids results in archimedean solids.
2. Describe the archimedean solids we obtain by rectifying each of the platonic solids.
3. What happens if you slide the planes of truncation even further?

Problem 7. Which of the archimedean solids we obtained so far can be truncated and rectified again? Describe the resulting archimedean solids. Can you truncate and or rectify those again?

There is one last method we can use to obtain archimedean solids:

Problem 8. Imagine an Icosahedron. At every vertex replace one of the triangles with a square or pentagon repespectively. This process is called snubbing.

1. Describe the resulting solids and argue why they are archimedean.
2. Does it make sense to replace the triangle with a hexagon, heptagon, etc.?
3. Does it make sense to snub any of the other platonic or archimedean solids.

Problem 9. Outline a possible strategy for proving that the 13 archimedean solids you found so far are indeed the only ones.