ORIGAMI CONSTRUCTIONS I

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In this problem set we will explore Origami construction. Such a construction is done by repeatedly folding and unfolding a piece of paper to imprint creases which we consider constructable. Hereby, it is not allowed to fold the paper more than once before completely unfolding it. We will show that anything constructable by with straight edge and compass is also constructable by folding.

Problem 1. Verify that you can indeed achieve the following basic constructions by folding. Can you think of any other ones?

(i) Intersect two lines.

(ii) Given two intersecting lines, bisect their angle.

(iii) Given two points P and Q, find the perpendicular intersector of PQ.

(iv) Given a point P and a line l, find the line perpendicular to l going through P.

(v) Given two points P and Q and a line l, find the line through Q that reflects P onto l.

(vi) Given two points P and Q and two lines l and l', find a line that reflects A onto l and B onto l'.

Problem 2. Show that the basic constructions 1-4 can be done using straight-edge and compass.

Problem 3. Which of the basic classical constructions can be easily done by folding? How should we think of circles.

Problem 4. Let l be a line and P be a point. Construct a parallel through P to l by folding.

Problem 5. Let a be a line segment and let l be a line and P be a point on l. Transfer the line segment a to l with start point P by folding. Be careful to distinguish three cases.

Problem 6. Let k be a circle around the point M containing the point A.

(i) Let l be a diameter of k. Use folding to determine the intersection of l and k.

(ii) Now let l be any line (close enough to M). Use folding to find the intersection points of k and l.

Problem 7. Let a and b be two given line segments. Construct a right triangle with legs of length a and b and a right triangle with hypotenuse a and one leg of length b.

Problem 8. Now we develop how we can intersect two circles by folding. Let k_1 and k_2 be two circles of radius b and c respectively. Assume their centers are a apart. Put k_1 around the origin of a coordinate system and k_2's center on the x-axis. Let l be the line defined by their intersection points.

(i) Draw a sketch.

(ii) Verify that the equation for the line l is

$$x = \frac{a^2 - b^2 + c^2}{2ab}.$$

(iii) Use right triangles to construct the length $\sqrt{a^2 - b^2 + c^2}$

(iv) Use similarity (or stretching) arguments to construct the length $\frac{a^2-b^2+c^2}{2ab}$.

(v) Use all of this to construct the intersection of two circles by folding.

Problem 9. Use folding to construct an equilateral triangle. Try using as little folds as possible.