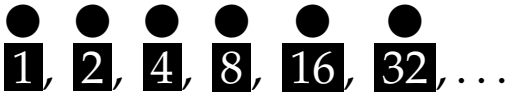
Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in kilograms):



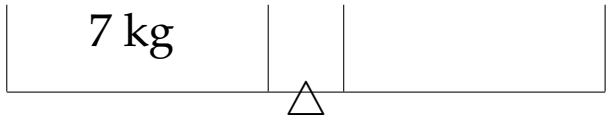
You can buy any number of these standard weights from the factory and use them with a balance scale.

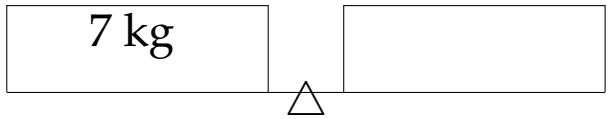
1. What is the pattern in the sequence of weights above?

2. Write down the next 3 weights in the sequence:

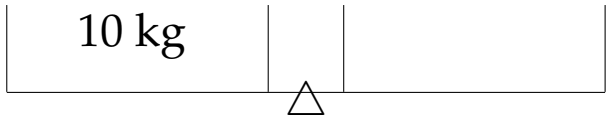
3. Balance each of the following objects using the weights
   ● ● ● ● ●
   1, 2, 4, 8, 16, ...

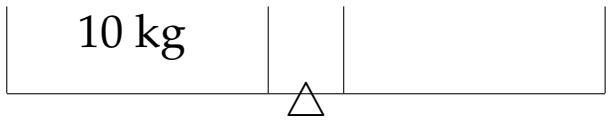
   (a) Find two different ways to balance a watermelon weighting 7 kilograms. (Note that you can use the same weight more than once).



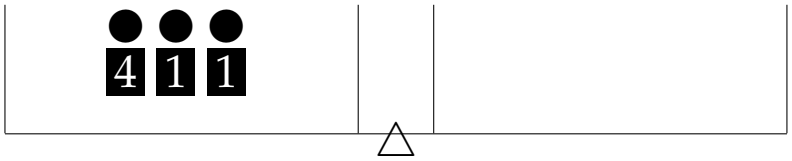


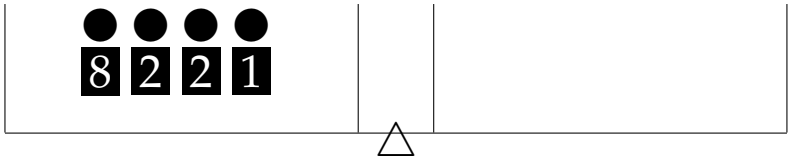
   (b) Find two different ways to balancs a metal ball weighing 10 kilograms. (Note that you can use the same weight more than once).

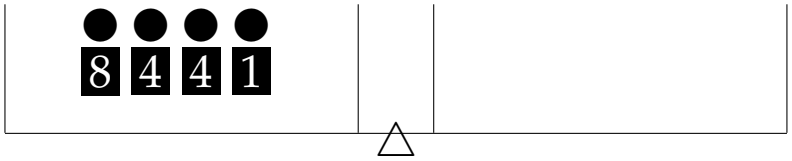


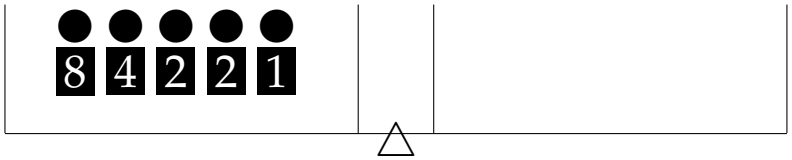


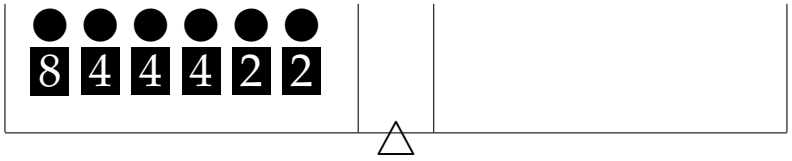
4. Now try to use each of the standard weight only once. Can you balance the following:

(a) 

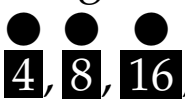
(b) 

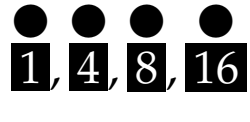
(c) 

(d) 

(e) 

5. Someone stole the 1 kilogram weight. What kind of weights can you balance now? (You still have all of the weights 
2, 4, 8, 16, . . .).

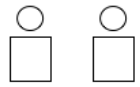
6. The next day, the 2 kilogram weight got stolen, too. Which weights can you balance now? (You have the weights 
4, 8, 16, . . .).

7. They found the 1 kilogram weight! Which weights can you balance now? (You have the weights 
1, 4, 8, 16 , . . .).

8. Balance the following. Do not use the same weight more than once. Fill in the table below:

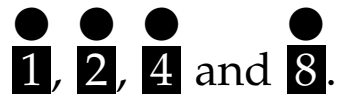
   - if you are using a certain weight, put 1 in its column;
   - if you are not using a certain weight, put 0 in its column.

   Some examples are filled in:

Weight	Write as sum of weights	8	4	2	1
1	$1 =$ 	0	0	0	1
2	$2 =$ 	0	0	1	0
3	$3 =$ 				
4	$4 =$ 	0	1	0	0
5	$5 =$				
6	$6 =$ 	0	1	1	0
7	$7 =$				
8	$8 =$				

Weight	Write as sum of weights	8	4	2	1
9	$9 =$ 8 1	1	0	0	1
10	$10 =$				
11	$11 =$				
12	$12 =$				
13	$13 =$ 8 4 1	1	1	0	1
14	$14 =$				
15	$15 =$				

9. Major robbery!

   (a) After many weights were stolen you are left only with the weights  , 2, 4 and 8. Can you balance 18 kilograms using each weight no more than once? Why or why not? What weights can you balance?

   (b) What is the largest number you can write as a sum of some the numbers $1, 2, 4, 8$ without using the same number more than once?

(c) Can you balance 30 kilograms with the given weights

$$1, 2, 4, 8, 16, \ldots$$

without using any weight more than once. That is, write 30 as a sum of several of these numbers, without repeating any of the numbers:

$$30 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 30. (Write 1 under the numbers that are used in the sum above. Write 0 under the numbers that are not used in the sum above):

	16	8	4	2	1
30					

(d) Can you write down 57 as sum of some of the numbers

$$1, 2, 4, 8, 16, 32, \ldots?$$

Do not use the same number twice.

$$57 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 57:

	32	16	8	4	2	1
57						

10. What number corresponds to the following strings of 0s and 1s. Fill in the table:

32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16+8+2=26$	26
0	1	1	0	1	0		
0	0	1	1	0	1		
1	1	0	0	1	0		
1	1	1	1	1	1		

11. Every number can be written in *binary notation:*

$$1=\boxed{1},\qquad 2=\boxed{1}\boxed{0},\qquad 3=\boxed{1}\boxed{1},\qquad 4=\boxed{1}\boxed{0}\boxed{0}.$$

Find the missing numbers or binary notation:

$$\boxed{1}\boxed{0}\boxed{1}\ =\qquad \boxed{1}\boxed{1}\boxed{0}=\qquad \boxed{1}\boxed{0}\boxed{0}\boxed{1}=$$
$$\square\square\square\ =\ 6\qquad \square\square\square=7\qquad \square\square\square=8$$

Here the numbers where each digit is written inside of a square are written in binary notation.