

“REVIEW” OF DIVISIBILITY AND COMBINATORICS

MATH CIRCLE (INTERMEDIATE) 4/8/2012

1) Suppose that $m = 5, n = 7, r = 20, s = 56, t = 210$. Answer the following:

a) Which of m, n, r, s, t are prime? Write of the prime factorizations of m, n, r, s, t .

b) Write of the prime factorizations of m, n, r, s, t .

c) What are the divisors of s ? Which of m, n, r, s, t are divisible by 7?

d) What is $\gcd(n, s)$? What about $\gcd(s, t)$?

e) What is $\text{lcm}(m, r)$? What about $\text{lcm}(r, s)$?

2) Prove that any natural numbers a and b satisfy the equation

$$\gcd(a, b) \cdot \text{lcm}(a, b) = a \cdot b.$$

3) Find all solutions in the natural numbers to the equation $x^2 - y^2 = 303$.

Recall that given any integer $q \geq 1$ we can write any number N as $N = k \cdot q + r$ where k, q, r are integers and $0 \leq r < q$. We say that r is the remainder when N is divided by q .

4) Prove that the number $n^3 + 2n$ is divisible by 3 for any natural number n .

5) Find the last digit in the number 2^{50} .

6) How many ways are there to rearrange the letters from the following words?

a) *COUNT*

b) *PIZZA*

c) *SEEDS*

d) *MISSISSIPPI*

7) Suppose there are 6 boys and 6 girls in a class.

a) How many ways are there for the students to be put in a line?

b) How many ways are there to put the students in a line with all the girls together?

c) How many ways are there to put the students in a line with all the girls NOT together?

d) How many ways are there to choose a President, Vice-President, Treasurer, and Secretary from the students?

e) How many ways are there to choose a group of size 4 from the students?

f) How many ways are there to divide the students into groups of size 3, 4, 5?

g) How many ways are there to divide the students into three groups of size 4?

Challenge 1) Find the last digit of 7^{7^7} .

Challenge 2) Repeat 7g) but ensure that no group consists entirely of boys or girls.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes