ORMC Olympiad Group
Winter: Week 7

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Problems

1. **(TJNMO-FR 2018)** Find all positive integer pairs (m, n) with $m \leq n$ so that $2^m + 2^n + 5$ is a perfect square.
2. Find all $k \geq 1$ so that
$$8k + 1 \mid 7k + 56$$
3. (a) Prove or disprove: if $a \equiv b \equiv 1 \pmod 5 \Rightarrow a^2 + b^2$ is not a square.
   (b) Prove or disprove: if $a \equiv b \equiv 1 \pmod 7 \Rightarrow a^2 + b^2$ is not a square.
4. (AIME 1988-modified) Find the smallest positive integer whose cube ends in 207.
5. **(TNMO-FR 2009-modified)** Find the largest positive integer which cannot be written as a sum of two positive composite numbers, one divisible by 7 and the other divisible by 13.
6. Find the number of divisors of 12! that gives remainder 1 when divided by 3.
7. Find minimum n so that $400 | 1 + 2 + \cdots + n$
8. **(TJNMO-FR 2012)** Find all positive integers n so that $7 \cdot 2^n + 1$ is a perfect square.

9. **(TJNMO-FR 2012-modified)** Find all positive integers n so that $5 \cdot 2^n + 9$ is a perfect square.

10. (TJNMO-FR 2009-modified) For how many positive integer pairs (x, y) the equality
$$xy^{19} = 2^{2022}(x-1)^{19}$$
holds?

11. **(AMC10 2016B P18-modified)** An increasing arithmetic sequence of positive integers is called 2-consecutive if the common difference is 2. In how many ways can 240 be written as the sum of an increasing sequence of two or more 2-consecutive positive integers?

12. Solve the following equations in positive integers:

    (a) $2^n - 3^m = 1$

    (b) $3^m - 2^n = 1$