Week 6: Hat puzzles

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Problem 1.
There are two contestants. They are given instructions by the host and allowed to confer before the game to come up with a strategy. At the start of the game, a hat that may be either blue or red is chosen at random from a large number of hats and placed on each contestant's head. As is standard, each contestant can see the other's hat but not her own. The host then sounds a buzzer, and each contestant must write the color of her hat on her own private tablet. If either of the contestants is correct, both of them win \$1,000. If there are 10 rounds, what is the maximum amount of money the contestants can expect to make? Can you explain how the strategy works?

Problem 2.
We have 3 hat colors and 6 prisoners.

a) Can they guarantee at least 2 correct guesses?

b) Can they guarantee at least 3 correct guesses? (Hint: use probabilities)

Problem 3.
a) Now we have 6 hats of different colors and 3 prisoners. Warden wears 3 of the hats on prisoners and hides the remaining hats. Prisoners can be sure that colors don't repeat. Can they guarantee at least 1 correct guess?

b) Same for 5 hats and 3 prisoners?

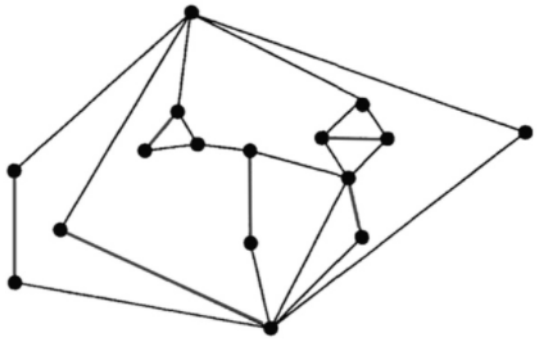
Problem 4.
Let wise men be arranged in a graph G. They can see only their neighbors. They wear either black or white hats. What is the maximal amount of correct guesses they can guarantee?

a) G is a complete graph on 4 vertices

b) G is a cycle on 4 vertices.

c) G is a complete bipartite graph $K_{1,3}$ as in the picture.

d) G is a graph like that



Problem 5 (For Advanced 3 only)**.**
In this variant there are 3 prisoners and 3 hats. Each prisoner is assigned a random hat, either red or blue. In all, there are three red hats and two blue. Each person can see the hats of two others, but not their own. On a cue, they each have to guess their own hat color or pass. They win release if at least one person guessed correctly and none guessed incorrectly (passing is neither correct nor incorrect).

This puzzle doesn't have a 100% winning strategy, so the question is: What is the best strategy? Which strategy has the highest probability of winning?