

Week 6: Hat puzzles

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Problem 1.

There are two contestants. They are given instructions by the host and allowed to confer before the game to come up with a strategy. At the start of the game, a hat that may be either blue or red is chosen at random from a large number of hats and placed on each contestant's head. As is standard, each contestant can see the other's hat but not her own. The host then sounds a buzzer, and each contestant must write the color of her hat on her own private tablet. If either of the contestants is correct, both of them win \$1,000. If there are 10 rounds, what is the maximum amount of money the contestants can expect to make? Can you explain how the strategy works?

Problem 2.

We have 3 hat colors and 6 prisoners.

- a) Can they guarantee at least 2 correct guesses?
- b) Can they guarantee at least 3 correct guesses? (Hint: use probabilities)

Problem 3.

- a) Now we have 6 hats of different colors and 3 prisoners. Warden wears 3 of the hats on prisoners and hides the remaining hats. Prisoners can be sure that colors don't repeat. Can they guarantee at least 1 correct guess?
- b) Same for 5 hats and 3 prisoners?

Problem 4.

Let wise men be arranged in a graph G . They can see only their neighbors. They wear either black or white hats. What is the maximal amount of correct guesses they can guarantee?

- a) G is a complete graph on 4 vertices
- b) G is a cycle on 4 vertices.
- c) G is a complete bipartite graph $K_{1,3}$ as in the picture.

