PROPOSITIONAL LOGIC

MATH CIRCLE (INTERMEDIATE) 3/11/2012

The language of propositional logic consists of:
propositional variables: $P, Q, R, \ldots$
connective symbols $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
and punctuation: $(,)$.
0) Write down the meaning of the connective symbols:

- $\neg$:
- $\wedge$:
- $\vee$:
- $\rightarrow$:
- $\leftrightarrow$:

Well-formed formulas, also called wffs, are defined so that:

- Every propositional variable is a wff.
- If θ and λ are wffs, then so are

$$(\neg\theta),\ (\theta \wedge \lambda),\ (\theta \vee \lambda),\ (\theta \rightarrow \lambda),\ (\theta \leftrightarrow \lambda)$$

For example,

$$P,\ (P \rightarrow Q),\ ((P \vee Q) \rightarrow R),\ ((\neg P) \wedge (Q \leftrightarrow (\neg R)))$$

are all all wffs, while NONE of

$$\rightarrow P, PQ \rightarrow R, (P \vee \wedge R, P \vee Q \rightarrow R$$

are wffs.

1) Translate the following statements into wffs (in propositional logic).

- It is raining.

- John has an umbrella.

- John is wet.

- If it is raining, then John has an umbrella.

- John is wet and it is raining.

- If it is raining and John does not have an umbrella, then he is wet.

- It is raining, but John does not have an umbrella, so he is wet.

- John is wet but it is not raining if and only if John has an umbrella or it is not raining.

2) Translate the following wffs into statements in English.

- P
- Q
- R
- $(P \wedge Q) \rightarrow \neg R$
- $P \leftrightarrow (Q \vee R)$
- $P \rightarrow (Q \rightarrow P)$
- $(\neg Q \vee P) \rightarrow R$

3) Complete the following table:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \to Q$	$P \leftrightarrow Q$
T	T					
T	F					
F	T					
F	F					

4) Suppose we know that P is T, Q is F, and R is T. (Note that this is called a *truth assignment* for P, Q, R.) What is the truth value of the following?

- $(P \wedge Q) \to R$
- $(\neg P \vee Q) \wedge (R \to Q)$
- $(Q \to (P \to R)) \leftrightarrow ((\neg Q \wedge P) \to R)$
- $[(\neg P \vee Q) \wedge (R \to Q)] \vee [(Q \to (P \to R)) \leftrightarrow ((Q \wedge P) \to R)]$

5) Translate the following into wffs, and then (using a truth table) decide if they are always true, always false, or neither.

- If there is smoke, then there is smoke.

- If there is smoke, then there is fire.

- There is smoke or there is fire, and there is not fire.

- If, if there is smoke there is fire, then there is smoke.

- John is big and dumb, and if John is big then he is not dumb.

- John is big and dumb, if and only if John is not big or not dumb.

6) An advertisement for a tennis magazine says: "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, then I'm reading about tennis." We can assume that the speaker can only do one of the above activities at a time. By translating the above into propositional logic, figure out what the speaker is doing.

Some problems are taken from:

- H. Enderton "A Mathematical Introduction to Logic"
- S. Russel, P. Norvig "Artificial Intelligence: A Modern Approach"