Week 4: Inscribed angles

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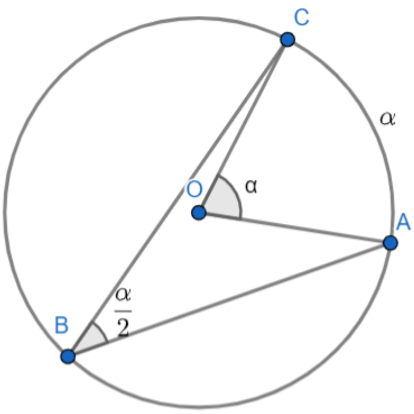
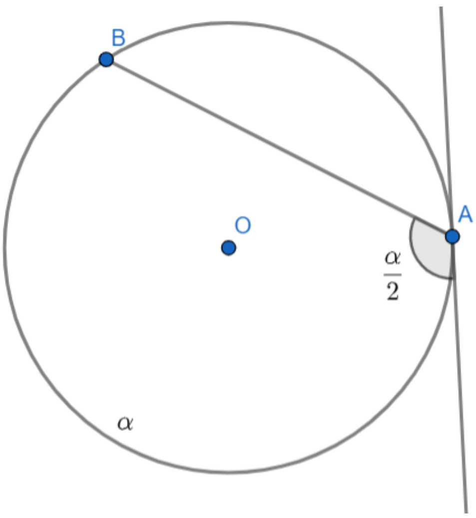
Definition 1.
The **angular measure** of an arc AB of the circle with center O is the measure of the **central** angle AOB containing the arc. For example, the angular measure of the semicircle is 180 degrees, and the angular measure of a quarter of the circle has 90 degrees.

Theorem 1.
Let O be the center of the circle. Then inscribed $\angle ABC = \frac{1}{2}\angle AOC$ if points B and O lie on the same side of AC, and $\angle ABC = 180^\circ - \frac{1}{2}\angle AOC$ if points B and O lie on opposite sides of AC.



Lemma 1.
The angle between chord AB and the tangent to the circle passing through point A is half the angular value of arc AB.



1 Problems

Problem 1.

a) From point A, which lies outside the circle, rays AB and AC go out, intersecting this circle. Prove that the value of the angle $\angle BAC$ is equal to the half-difference of the angular values of the circular arcs enclosed within this angle.

b) Draw two lines through any point P inside a circle. Name the points where these lines intersect the circle A, B, C, and D, clockwise. Show that angle APB is equal to half the sum of arcs AB and CD.

Problem 2.

From point P, located inside the acute angle BAC, perpendiculars PC_1 and PB_1 are drawn to straight lines AB and AC. Prove that $\angle C_1AP = \angle C_1B_1P$.

Problem 3.

Prove that all the angles formed by the sides and diagonals of a regular n-gon are integer multiples of $180°/n$.

Problem 4.

The bisector of the outer angle at the vertex C of triangle ABC intersects the circumcircle at point D. Prove that $AD = BD$.

Problem 5.

The incircle center of triangle ABC is symmetrical to the circumcircle center about side AB. Find the angles of triangle ABC.

Problem 6.

The vertex A of an acute-angled triangle ABC is connected by a line segment to the center O of the circumcircle. The height AH is drawn from the vertex A. Prove that $\angle BAH = \angle OAC$.

Problem 7.

Prove that given a triangle ABC and a point P on its circumcircle, the three closest points to P on lines AB, AC, and BC are collinear.