INDUCTION I

MATH CIRCLE (INTERMEDIATE) 2/19/2012

1) Simplify the sum $1+2+3+\cdots+n$. (Try to come up with a way to argue your answer is correct if you can!)

2) Simplify the sum $1\cdot 2+2\cdot 3+\cdots+(n-1)\cdot n$.

The Method of Mathematical Induction (MMI) is a way to show that a property $P(n)$ holds for all $n = 1, 2, 3, \ldots$. A proof using MMI consists of two steps:

Basis: Show that $P(1)$ holds.

Induction: Assume $P(k)$ holds, and show that $P(k+1)$ holds.

Then, MMI tells us that $P(n)$ holds for all n.

3) Using MMI, prove your answers for 1) and 2).

4) Show that $1+3+5+\cdots+(2n-1)=n^2$. For an added challenge (after you have done the proof using MMI), see if you can come up with a proof using only the equation proved in 1).

5) Show that for all $n \geq 1$, $3^n > n^2$.

6) Show that $8^n - 1$ is always divisible by 7. Recall we write this as $7|8^n - 1$.

7) Show that $9|4^n + 15n - 1$.

Challenge 1) Prove or disprove: $n^2 + n + 41$ is prime for all n.

Challenge 2) The natural numbers have the "well order principle": any non-empty set (of natural numbers) has a least element. Prove MMI using the well order principle. (Hint: Use a proof by contradiction.)

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes