

Cryptarithms

Cryptarithmic, also known as cryptarithm, alphametics, or word addition, is a math game of figuring out unknown numbers represented by words. Different letters correspond to different digits. Same letters correspond to same digits. The first digit of a number cannot be zero.

Problem 1 *Solve the following cryptarithm.*

$$\begin{array}{r} H E \\ + H E \\ H E \\ \hline S H E \end{array}$$

Looking at the fourth column, we see that O can be either zero or one. In the latter case, one is carried over from the third column, so $E + 1 \geq 9$. $E \neq 9$ because $S = 9$, so $E = 8$. If this is the case, then one is carried over from the second column and $N = 0$. In the second column, this gives $0 + R = 10 + E = 18$, so $R = 18$ and is a single digit number at the same time. Thus, $O = 0$ and $E = N - 1$.

$$\begin{array}{r}
 9 \ N-1 \ N \ D \\
 + \ 1 \ 0 \ R \ N-1 \\
 \hline
 1 \ 0 \ N \ N-1 \ Y
 \end{array}$$

Suppose that one is not carried over from the first column to the second. Then $N + R = 10 + N - 1$ implying $R = 9$, an impossibility since $S = 9$. Thus, one is carried over from the first column and $N + R + 1 = 10 + N - 1$, giving us $R = 8$ and $D + N - 1 \geq 10$.

$$\begin{array}{r}
 9 \ N-1 \ N \ D \\
 + \ 1 \ 0 \ 8 \ N-1 \\
 \hline
 1 \ 0 \ N \ N-1 \ Y
 \end{array}$$

Since $Y \neq 0, 1$, the latter inequality can be strengthened to $D + N > 12$. The single-digit number N can only take values 3, 4, 5, 6, 7. Let us consider these possibilities case by case.

$N = 3$ implies $D > 9$.

$N = 4$ implies $D > 8$, but $D \neq 9$.

$N = 5$ implies $D > 7$, but $D \neq 8, 9$.

$N = 7$ implies $D > 5$, so $D = 6$ and $N - 1 = 6$.

Finally, $N = 6$ implies $D > 6$, so $D = 7$. $7 + 5 = 12$, so $Y = 2$. Here comes the solution.

$$\begin{array}{r} 9567 \\ + 1085 \\ \hline 10652 \end{array}$$

Problem 2 *Solve the following cryptarithm.*

$$\begin{array}{r} F O R T Y \\ + \quad \quad T E N \\ \quad \quad \quad T E N \\ \hline S I X T Y \end{array}$$